**Easy Round:** 30 seconds

**E1** Suppose $x$ and $y$ are positive integers satisfying $x^3y + 35y^4 = 2018$. Find $xy$. [18]

**E2** Let $A$, $B$, and $C$ be digits of base 7 numbers, with possible values $0, \ldots, 6$. If $\text{ACB}_7 + \text{BCC}_7 = 1400_7$, what is the base-10 value of $\text{ACB}_7$? [319]

**E3** **a** What is the remainder when $17^{17^{17}}$ is divided by 7? [5]

**b** What is the largest 2-digit prime factor of $\binom{200}{100}$? [61]

**E4** Every day Jill leaves home to pick up her husband, Jack, at the train station. They arrive simultaneously at 5:00 AM and immediately drive home together. Jill always drives at a constant speed. However, one day, Jack catches an earlier train and arrives at 4:00 PM. He then starts to walk home. On the way, he meets Jill, who is on her way to meet the 5:00 PM train. He gets into the car and they arrive home 20 minutes earlier than usual. How many minutes had Jack been walking? [50 minutes]

**E5** In a classroom, there are 47 students in 6 rows and 8 columns. Every student's position is expressed by $(i, j)$. After moving the position changes to $(m, n)$. Define the *change* of every student as $(i - m) + (j - n)$. Find the maximum of the sum of changes of all students. [12]

**E6** You are asked to go to the store and purchase apples, bananas, and oranges. Your mom says to buy a total of 20 pieces of fruit and your brother says that you must buy at least 1 of each fruit. How many ways can you make this purchase? [171]

**E7** **a** If $x > y > 0$ and $2\log_{10}(x - y) = \log_{10} x + \log_{10} y$, what is the value of $\frac{x}{y}$? $\left[\frac{3+\sqrt{5}}{2}\right]$

**b** Let $N$ be a 3-digit positive number with distinct nonzero digits. What is the smallest possible value for the ratio of $N$ to the sum of its digits? [10.5]

**E8** **a** There are 46 656 six-digit numbers that can be formed from the digits 1, 2, 3, 4, 5, and 6, with repetition of digits allowed. If these numbers are listed in order, what is the 2018th number on the list? [1 234 312]

**b** What is the solution set of the inequality $\frac{1}{x} + 2x \geq 3$? $\left[\left(0, \frac{1}{2}\right] \cup [1, \infty)\right]$

**E9** A point inside a circle of radius $\sqrt{50}$ lies 2 units directly below a point on the circle, and 6 units directly to the right of a point on the circle, and 6 units directly to the right of a point on the circle. What is the distance from the center of the circle to this point? $[\sqrt{26}]$

**E10** Alice and Bill live at opposite ends of the same street. They leave their houses at the same time and each walk, at a constant speed, from their house to the other house and back. The first time they meet, they are 400 meters from Alice's house, and the second time they meet, they are 300 meters from Bill's house. Both times they are travelling in opposite directions. What is the distance, in meters, between the two houses? [900]

**Average Round:** 60 seconds

**A1** You roll a fair die five times and add the numbers that come up. What is the probability that the sum is 10? $\left[\frac{7}{432}\right]$

**A2** Eight circles of radius 1 have centers on a larger common circle and adjacent circles are tangent. Find the area of the common circle. $[2\pi(2+\sqrt{2})]$

**A3** **a** Find the sum of the coefficients of $(x+1)(x+2)(x+3)\cdots(x+2017)(x+2018)$ when completely expanded. [2019!]

**b** Let $Z$ and $D$ be points on sides $AC$ and $CB$ of triangle $ABC$. If $\angle BDA = \angle ZDA = 60°$, $\angle DAB = 80°$, and $\angle ACB = 10°$, find the answer.* $[40°]$

**A4** A square is inscribed in a circle, and the region inside the circle but outside the square is shaded. A circle is then inscribed in the square, and the second square is inscribed inside the second circle. The region inside the second circle but outside the second square is shaded. If this process is continued until infinity, what fraction of the area of the original circle is shaded? $\left[2 - \frac{4}{\pi}\right]$

**A5** Find the area of the figure. Angles are in degrees. $[18 \text{ units}^2]$

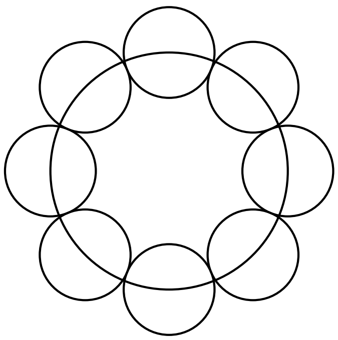

Figure for **A2**

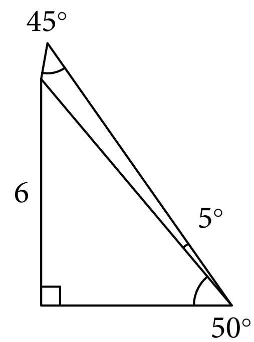


Figure for **A5**

**Difficult Round:** 90 seconds

**D1** You have 17 apples and 7 friends, and you want to distribute apples to your friends. The only requirement is that Steven, one of your friends, does not receive more than hald of the apples. Given that the apples are indistinguishable, and friends are distinguishable, compute the number of ways that the apples can be distributed. $[97\,944]$

**D2** **a** Find all positive integers $k$ for which there does not exist a positive integer $a$ such that $a(a+k)$ is a perfect square. $[1, 2, 4]$

**b** Find the radius of the largest circle that lies above the $x$-axis and below the parabola $y = 2 - x^2$. $\left[\frac{2\sqrt{2}-1}{2}\right]$

**D3** Find the number of solutions in positive integers to the equation $5x^2 + 4y^2 + 12yz + 2xz = 2019$. $[0]$

**D4** Three points are independently chosen at random on a circle. What is the probability that they form an acute triangle? $\left[\frac{3}{4}\right]$

**D5** **a** How many 3-element subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ are there for which the sum of the elements in the subset is a multiple of 3? $[57]$

**b** Given 4 random points on a sphere, find the probability that the tetrahedron formed by these 4 points contains the sphere's center. $\left[\frac{1}{8}\right]$

* Voided.