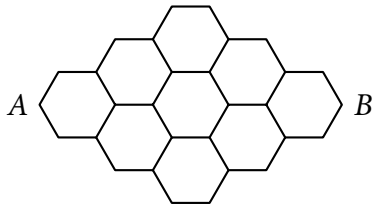
Easy

E1 Solve for all $x \in [0, 2\pi]$ such that $2\cos 3x \sin 2x = \cos 3x$.
$\left[\left\{\frac{\pi}{12}, \frac{\pi}{6}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{12}, \frac{3\pi}{2}, \frac{11\pi}{6}, \frac{17\pi}{12}\right\}\right]$

E2 Find the rightmost nonzero digit of 23!. [4]

E3 Find the last two digits of $2011^{2010^{2009^{\cdots^{2^1}}}}$. [01]

E4 In how many ways can you go from A to B if you can only move in the following directions: $\nearrow$, $\searrow$, $\rightarrow$?
[20]



E5 In how many ways can 7 people sit in a row if two particular people Jhonah and Vicman must be at least two seats apart? [2400]

E6 Convert the octal number $(7514)_8$ to binary. $[(111\,101\,001\,100)_2]$

E7 Let u and h be integers. Find the values of u and h such that $3(580 + u)^2 = 1\,0h0\,763$. $[u = 9, h = 4]$

E8 A farmer needs to take his goat, wolf and cabbage across the river. His boat can only accommodate him and either his goat, wolf or cabbage. If he takes the cabbage, the wolf will eat the goat. If he takes the wolf, the goat will eat the cabbage. Only when the man is present are the cabbage and goat safe from their respective predators. Taking the man, man and goat, man and the wolf, and man and the cabbage across the river cost ₱5, ₱8.5, ₱9.5, ₱6, respectively. What is the minimum amount the farmer needs to pay to get everything across the river? [₱51]

E9 Find a function $f(X)$ defined on $\mathbb{R}$ such that $f(x + 2) = 9f(x)$ for all values of x.
$[f(x) = k3^x$, where $k \in \mathbb{R}]$

E10 From port P, ship A is 5 km away 15° north of east, and ship B is 6 km away 30° west of north. What is the distance between the two ships? $\left[\sqrt{61 - 15\sqrt{2} + 15\sqrt{6}}\right]$

E11 Let ABC be a triangle with $AB = 6$, $BC = 4$, $AC = 8$. Let P, Q, R be the midpoints of BC, AC, AB respectively and S be the intersection of AP, BQ, CR. Let $T = AP \cap QR$. Find the length of ST.
$\left[\frac{\sqrt{46}}{6}\right]$

E12 If $M + S + A = MSA \neq 0$ and $X = \frac{(1 - M^2)(1 - S^2)}{MS} + \frac{(1 - S^2)(1 - A^2)}{SA} + \frac{(1 - A^2)(1 - M^2)}{AM}$, find the numerical value of X. [4]

E13 Quadrilateral $ABCD$ is inscribed in a circle with $DA = 5$, $AB = 5$, $BC = 7$. If $\triangle ABD$ is right, find the distance between points A and C. $[4\sqrt{2}]$

E14 Alvin has 5 identical red beads and 7 identical blue beads. He wants to create a bracelet by lacing a string through the 12 beads, with no two red beads beside each other, and tying the ends of the string. Out of all distinct bracelets that he can make, what fraction has 3 blue beads placed beside each other? $[\frac{1}{3}]$

E15 A box contains 23 identical black socks and 23 identical gray socks. A separate drawer contains a pile of black socks. Joseph takes out two socks from the box. If he drew a matching pair, he puts a black sock from the drawer to the box; if not, the gray sock drawn is returned to the box. Jeysen continues this process until the last two socks are taken out and the last sock is put in the box. What is the probability that the last sock is gray? [1]

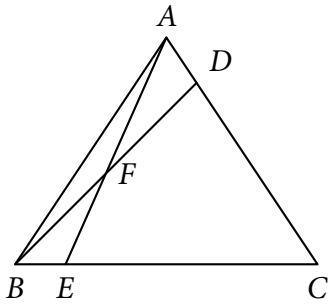
Average

A1 Sixty-four points (a, b) are selected in the first quadrant of the Cartesian plane such that $a, b \in \mathbb{N}$, $a \leq 8$ and $b \leq 8$. Two distinct points are selected from the 64 points. What is the probability that the two points are contained in the same vertical or horizontal line, or in a line with slope 1 or -1? $\left[\frac{13}{36}\right]$

A2 On a circle with radius 1, two points are chosen randomly from the circumference. What is the probability that the distance between the two points is less than 1? $\left[\frac{1}{3}\right]$

A3 Find the remainder when $\dfrac{45!}{23}$ is divided by 23. [1]

A4 In the figure, $AF : FE = 3 : 2$ and $BF = FD$. If the area of $\triangle ABC$ is 60 square units, what is the area of $CEFD$? [44 square units]



A5 What is the maximum number of points of intersection of 25 distinct congruent equilateral triangles in a plane. [1800]

A6 Let a and b the roots of the equation $x^2 + 5x + 9 = 0$. Find the value of $\left(\dfrac{a}{b+3}\right)^2 + \left(\dfrac{b}{a+3}\right)^2$. $\left[\frac{10}{9}\right]$

A7 Determine all possible values of x that satisfy the equation $\dfrac{3+x}{3x} = \sqrt{\dfrac{1}{9} + \dfrac{1}{x}\sqrt{\dfrac{4}{9} + \dfrac{2}{x^2}}}$. $\left[\frac{3}{4}\right]$

A8 Jed and Leah met each other for the first time on December 18, 2008. Find the exact date such that they have known each other for exactly N days, where N is the year of that date. [June 24, 2014]

A9 The point $A(4, 4)$ is rotated 60° counterclockwise about the origin to a point B. Find B. $\left[(-2\sqrt{3}+2, 2\sqrt{3}+2)\right]$

A10 Let A and B be distinct decimal digits. Find all possible values for the two digit number AB such that the numbers 33AB6, 19AB, and 34A B71 have a common divisor greater than 1. [27 and 74]

Difficult

D1 Terrence, Ramon, Leah, Heart, Mayen, Achel and May all have different weights. They are to be seated in a row of 7 chairs. Because all of them are insecure, no one wants to sir between two people who are both lighter than him or her. How many possible seating arrangements are there? [64]

D2 In base 26, the letters A to Z correspond to the numbers 0 to 25 respectively. Divide $FOUR_{26}$ by TWO_{26}. (Give the quotient and the remainder.) [H_{26} remainder FSX_{26}]

D3 An infinite periodic continued fraction is a number that is of the form $a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\ddots}}}}$ where $a_{k+m} = a_k$ for some m. Express $\sqrt{7}$ as an infinite periodic continued fraction. $\left[2 + \cfrac{3}{2 + \cfrac{3}{2 + \cdots}}\right]$

D4 If the roots of $f(x) = 2x^3 - 39x^2 + 95x + 2$ are a, b and c, find the value of $\left(1 - \frac{1}{a^2}\right)\left(1 - \frac{1}{b^2}\right)\left(1 - \frac{1}{c^2}\right)$. [$-2010$]

D5 A square piece of paper $ABCX$ is folded in the following manner: Point X is first folded onto point A. Then, point X is folded onto various points X' on the edge $\overline{AB}$, and finally, X is folded onto point B. After folding X continuously to points X' from A to B, a curve determined by the creases runs from the midpoint of $\overline{AX}$ until point C. If we let A be the origin and X be the point $(0, -1)$, find the equation of this curve. Assume the domain is $[0, 1]$. [$y = -\frac{1}{2}x^2 - \frac{1}{2}$]