Survival Round

Easy

SR-E1 Find the smallest positive integer r such that $12! \cdot r$ is a perfect square. [231]

SR-E2 Find the value of the product

$$\left(1+\log_{10}\left(1+\frac{1}{10}\right)\right)\left(1+\log_{11}\left(1+\frac{1}{11}\right)\right)\left(1+\log_{12}\left(1+\frac{1}{12}\right)\right)\cdots\left(1+\log_{99\,999}\left(1+\frac{1}{99999}\right)\right).$$

[5]

SR-E3 The number of possible groups of at least one person that Alec's Minions can form is 192 more than that of Mojo's Jojos. If a Minion cannot be a Jojo and vice versa, how many people are there in Alec's Minions? [8]

SR-E4 For what values of a and b would make the six-digit integer $20a\,b11$ divisible by 33? [$(a,b) = (0,2),(3,5),(6,8)$]

SR-E5 For some real number k, the quadratic equation $x^2 + 2kx + k^2 - 1$ has both roots lie on the interval $[-2,4]$. What must be the value of k? [$k \in [-1,3]$]

SR-E6 A lattice point in the Cartesian plane is a point (x,y) such that $x, y \in \mathbb{Z}$. How many lattice points are there on, and in the interior of, the circle $x^2 + y^2 = 25$? [81]

SR-E7 It is Roxanne's lunch time so she went to a restaurant to eat. Upon arriving at the restaurant, she noticed that they serve 7 different appetizers, 7 types of main courses, 7 kinds of drinks, and 7 sweet desserts. She gets at most one of each type and she needs to have a main course and a drink. If she also has an option of getting house water instead of a drink, how many different combinations can she order? [3584]

SR-E8 Given that $\sqrt[4]{x} - \frac{1}{\sqrt[4]{x}} = 4$, find the real value of $\sqrt[6]{x} + \frac{1}{\sqrt[6]{x}}$. [3]

SR-E9 Cards numbered 1 to 24 are arranged in a deck in numerical order; that is 1 is on top and 24 is at the bottom. The top card is placed on the table and the next card is transferred to the bottom of the deck. Now the top card is placed on top of the card on the table and next card is transferred at the bottom of the remaining deck. The process is repeated until only one card remains. What is the number of the only card remaining on the deck? [16]

SR-E10 Find all positive integer solutions (a,b,c) of the system $\begin{cases} a+b+c=8 \\ a+5b+10c=34 \end{cases}$. [$(4,2,2)$]

Average

SR-A1 Solve for all possible values of y: $\sqrt{y^2+4y+8} + \sqrt{y^2+4y+4} = \sqrt{2(y^2+4y+6)}$. [$y = -2$]

SR-A2 Given that $f(x) = \dfrac{(x^4-1)(x-4)}{(x^2-1)^{3/2}}$ and $h(x)$ are functions on the set of real numbers, determine the y-intercept of $h(x)$ if $(h \circ f)(x) = \dfrac{x^2-8x+13}{x}$. $\left[-\frac{3}{4}\right]$

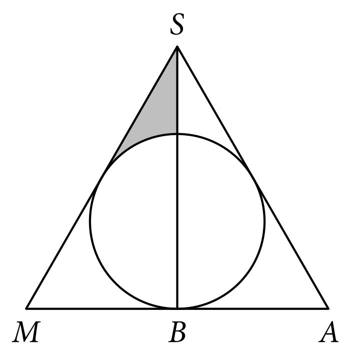
SR-A3 If $x = \sqrt{2011 + \sqrt{2011 + \sqrt{2011 + \sqrt{\cdots + \sqrt{2011}}}}}$, where the number 2011 appears 2011 times, what is the value of $\lfloor x \rfloor$? [45]

SR-A4 Two friends, BJ and Jeng, wanted to do their homework together. Unfortunately, BJ only does homework between 3 AM and 4 AM while Jeng only starts doing his homework when the clock's minute hand and hour hand form an angle of 180° with each other. Assuming the hour and minute hands move every second, at what time (rounded up to the nearest second) do they start doing their homework?
[3 : 49 : 06]

SR-A5 A polynomial $f(x)$ leaves a remainder of -4 when divided by $x+2$, leaves a remainder of 2 when divided by $x-1$, and leaves a remainder of 2 when divided by $x+1$. Find the remainder when $f(x)$ is divided by $(x+2)(x^2-1)$.
$[-2x^2+4]$

SR-A6 Find all ordered triples (x, y, z) such that $x+y+z = 91$, $1 < x < y < z$ and x, y, z form an increasing geometric progression of positive integers.
$[(13, 26, 52), (7, 21, 63)]$

SR-A7 Circle C is inscribed in equilateral triangle MSA, as shown in the figure. SB is one of the triangle's altitudes. If the area of the shaded region is $(3\sqrt{3}-\pi)$ square units, what is the length of one side of the triangle?
$[6\sqrt{2}$ units$]$



SR-A8 Dale has seven beautiful suitors, namely Lou, Pamela, Angela, Kathrina, Jeckah, Venice, and Maddy. Each suitor is uniquely scheduled on a specific day of the week to send him love letters, but they should send their letters anonymously. However, Prince Dale knows three things:

- Pamela and Angela can send letters only during weekends.
- Lou and Jeckah, who do not send letters on Mondays, send letters three days after Venice and Maddy sent letters, respectively.
- Kathrina does not send her letters a day before Lou, while Pamela does not send hers a day before and after Venice.

Which suitor does Dale want to marry if he wants the one who sends letters during Tuesdays?
[Venice]

SR-A9 A real number x is said to be a *TeaM Energy number* if it is a solution to the equation $2n|x| - n(-1)^x = 2011$ for positive integers $n \le 2011$. Determine all possible TeaM Energy numbers.
$[\pm 1005, \pm 1006]$

SR-A10 Hana hates repeating digits in a number. So when she counts, she skips those numbers whose digits are not distinct. Suppose that after counting all the books sold during a book sale, she "counted" that there are exactly 1028 books sold. To her surprise, it exceeded the initial number of books, and finally remembered that she counts in an odd manner. How many books were actually sold?
[744]

Difficult

SR-D1 In an unreleased music video, Jagger, Christina and Adam, form a straight line. Christina, who is in the middle of the line, is 2011 inches away from both Jagger and Adam. At the start of the song, Jagger and Adam walk forward at a constant speed. Christina starts walking to Jagger. Upon reaching Jagger, she walks back to Adam. Upon reaching Adam, she walks backs to Jagger, but the song ends by the time she is 2011 inches away from him. If Jagger moved a total of 2011 inches during the song, how far did Christina walk? $\left[2011(1+\sqrt{2})\text{ inches}\right]$

SR-D2 Points X and Y are chosen in the interior of sector AOB with center O and endpoints A, B. If the radius of sector AOB is 2011 cm and $m\angle AOB = 90°$, find the probability that $m\angle XOY \geq 30°$. $\left[\frac{4}{9}\right]$

SR-D3 Given $z \in \mathbb{C}$, find the zeros of the complex polynomial $P(z) = z^2 - 2z + (-2+4i)$. $[3-i, -1+i]$

SR-D4 Evaluate $\left\lfloor \frac{2!+1!}{1!+0!} \right\rfloor + \left\lfloor \frac{3!+2!}{2!+1!} \right\rfloor + \left\lfloor \frac{4!+3!}{3!+2!} \right\rfloor + \cdots + \left\lfloor \frac{2011!+2010!}{2010!+2009!} \right\rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. $[2\,021\,055]$

SR-D5 Desiree and MG are playing with an old robot. After some time, they notice that the largest number the robot can represent is $2^8 - 1 = 255$. Any operation that results in a number greater than the maximum *overflows*, e.g. $(2^8-1)+1 = 0$, $(2^8-1)+2 = 1$. They then thought of playing a game. First, the robot randomly generates any positive integer that is at most 255. Desiree and MG then take turns choosing a number in the set $\{2, 4\}$ which is then multiplied to the number that the robot has. The first player that lets the robot make a wrong computation through overflow loses the game. If Desiree makes the first move, and both players play the optimal strategy, what is the probability that Desiree wins? $\left[\frac{109}{255}\right]$

SR-D6 Find y such that there is a real number x which satisfies the following equation $\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}} = y = x + \frac{1}{x+}\frac{1}{x+}\cdots$. $\left[\frac{1+\sqrt{5}}{2}\right]$

SR-D7 May has the habit of writing consecutive positive integers, starting from 1, on a clean sheet of paper. After writing the integer k, she gives the paper to Jason. Jason, on the other hand, starting from 1, counts all 1's that appear in the sequence of digits that May wrote, and declares that there are exactly 2011 1's on the paper. What is the value of k? $[3172]$

Quaterfinal Round

Easy

QFR-E1 Find the degree measure of the smaller angle formed by the hour and minute hand of an analog clock at 3:15 PM. $[7.5°]$

QFR-E2 Starting from the point $(0,1)$, Dumie runs around a circular track centered at the origin in a clockwise direction at constant speed. Ten minutes after leaving the start point, the product of his x and y coordinates was $-\frac{\sqrt{2}}{3}$. What will be his y-coordinate after another ten minutes? $\left[-\frac{2\sqrt{2}}{3}\right]$

QFR-E3 Let $f(x) = a_1x + a_2x^2 + \cdots + a_{2010}x^{2010}$ and $f(k) = -kf(k-1)$ for $k = 1, 2, \ldots, 2011$. Find $f(2011)$. $[0]$

QFR-E4 Let E, G, I, L, N, O, W represent distinct unique digits. What does $WOWOWEE$ represent if $WILLING - WILLIE = WOWOWEE$? [5 050 522]

QFR-E5 Determine the value of the infinite product $\left(1+\frac{3}{7}\right)\left(1+\left(\frac{3}{7}\right)^2\right)\left(1+\left(\frac{3}{7}\right)^3\right)\left(1+\left(\frac{3}{7}\right)^4\right)\cdots$.

$\left[\frac{7}{4}\right]$

QFR-E6 What is the sum of the reciprocals of the roots of $x^3 - 5x + 2$? $\left[\frac{5}{2}\right]$

Average

QFR-A1 Jackie is mixing milk and tea with the help of her infinite army of monkeys. Every monkey holds a cup of milk tea, and each monkey beginning with the second holds $\frac{2}{3}$ as much milk tea which is $\frac{3}{5}$ as milk-concentrated as that of the last monkey. She mixes the contents of all these cups to obtain 500 mL of a 50% milk and 50% tea solution. Find the volume of milk tea and the concentration of tea in the first cup. $\left[\frac{1}{6}\text{ L}, \frac{1}{10}\text{ tea-concentration}\right]$

QFR-A2 If $T_n = 1!\cdot(1^2+1+1) + 2!\cdot(2^2+2+1) + \cdots + n!\cdot(n^2+n+1)$ for any nonnegative integer n, what is the value of $\frac{T_{2011}+1}{2012!}$? [2012!]

QFR-A3 One rainy day, Geldof arrived at school very late that he missed his morning class. Looking at the board in the classroom, he sees that a polynomial is given as a homework, but he can only decipher $x^7 - 14x^6 - 42x^5 +$, because the professor erased almost everything. Nadine told him that the seven real roots of this polynomial form an arithmetic sequence. Without knowing the entire polynomial, Geldof was able to find all its roots. What is the largest root of this polynomial?

[11]

QFR-A4 A function f on the set of positive integers operates in the following manner: If n is a one-digit number, $f(n)$ is given by the units digit of n^3. Else, starting from the leftmost digit of n, if $f(a_i) \neq a_i$, f replaces a_i with $f(a_i)$. If $f(a_i) = a_i$, f is performed on the next digit to the right, a_{i+1}, instead. Finally, for an arbitrary integer c, $f(c) = c$ if $f(a_n) = a_n$, where a_n is the last digit of c. Find the largest possible four-digit integer j with distinct digits such that $f(f(f(f(f(j))))) = f(f(f(f(j)))) \neq f(f(f(j)))$. [9864]

QFR-A5 Papa Goma wants to make an unusual quadrilateral picture frame form Mama Mary's lovely picture. Using a reference coordinate system, Papa Goma placed the vertices of the frame at $(1,8)$, $(10,6)$, $(0,2)$, and at $(m,0)$, where $m > 0$. Because Mama Mary's picture is very large, he needs the frame to have an area of 48 square units. What must be the value of m? [$m = 5$]

Difficult

QFR-D1 A new chess piece, the *robot unicorn*, is introduced. In a single move, it can do any one of the following:

- Move 1 square up then 2 squares right.
- Move 2 squares up then 1 square right.

If the robot unicorn starts from the lower-left corner of a 12×14 chess board, in how many ways can it go to the upper-right corner? [56]

QFR-D2 Define the operation $a * b$ on real numbers a and b by the following: $a * b = a + \cfrac{b}{a + \cfrac{b}{a + \cdots}}$. Solve for x in $x * (-2011) = 2011$. [2012]

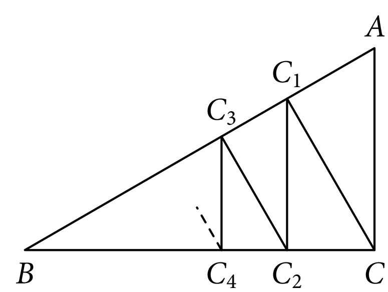
QFR-D3 Find the last digit of $2011^{2010} - 2010^{2009} + 2009^{2008} - \cdots - 2^1 + 1^0$. [2]

QFR-D4 A game is played where people bet on the sum of two 6-sided dice that will be rolled. However, Jeffrey loads the dice such that the number 1 is n times more likely to occur than the other 5 faces, all of which are equally likely to appear. If he wanted the sum of 2 to appear most often, what should the minimum value of n be? [4]

Semifinal Round

Easy

SFR-E1 Given a right triangle ABC with $BC = 1$ and $\angle C = 90°$, perpendiculars are drawn starting from C to a point C_1 on hypotenuse $\overline{AB}$ then from C_1 to a point C_2 on leg $\overline{AC}$, and then back to hypotenuse $\overline{AB}$ at point C_3 and so on. If the total length of the perpendiculars drawn is 1 unit, find the measure of $\angle B$. $\left[\theta = \frac{\pi}{6}\right]$



SFR-E2 Starting on a certain date, Lovely and Ramon decided to meet only on prime-numbered days p of each month for p minutes. If as of February 14, 2011, they have met for exactly 143 minutes, on what date was their first meeting? [January 19, 2011]

SFR-E3 Philip rolls a fair 6-sided die and Jared rolls 2 fair 6-sided dice. Find the probability that the sum of Jared's rolls equals the roll of Philip. $\left[\frac{5}{72}\right]$

SFR-E4 Snack stackers Cantor and Janina can stack snacks in a snack stack in 3 hours and 10 hours, respectively, and can snack stacked snacks at a rate of 5 snacks per hour and 1 snack per hour, respectively. If a snack stack can hold 150 snacks, how long could the snack stackers stack snacks in a snack stack if the snack stackers snack stacked snacks? Express your answers in hours. $\left[\frac{150}{59} \text{ hours}\right]$

SFR-E5 Find the sum of solutions of the equation $\left(\frac{x+6}{x-3}\right) - 20\left(\frac{x-3}{x+6}\right) = 8$. [4]

Average

SFR-A1 A certain polyhedron is made by cutting congruent triangular pyramids from each of the vertices of a regular tetrahedron with volume 81 in^3. If four of the faces are regular hexagons, find the volume of this polyhedron. [69 in^3]

SFR-A2 Janina made a robot that tells you if a positive integer is prime. During her nightly sleepwalking, she unintentionally edited the configuration of the robot that if the integer is odd, the probability that it will tell the correct output is 69% and if the input is even, the probability that it will tell the correct output is 25%. Janina tests the robot by trying two random integers greater than 1. What is the probability that at least one output is correct? [0.7191]

SFR-A3 McCarthy's famous function, $f : \mathbb{N} \mapsto \mathbb{N}$, is defined as $f(x) = \begin{cases} f(f(x+11)), & x \leq 100 \\ x - 10, & x \geq 101 \end{cases}$. Find $\sum_{n=1}^{100} f(n)$. [9100]

SFR-A4 In a certain gameshow, the contestant loses whenever she makes three mistakes. If each question has four choices and the contestant randomly guesses each time, what is the probability that she loses after answering the fifth question? $\left[\frac{81}{512}\right]$

Difficult

SFR-D1 For what values of $x \in (0, 2\pi)$ is $\sin(\lfloor x \rfloor - 1) + \cos\lfloor x \rfloor \sin(1) + 2\sin\lfloor x \rfloor \sin^2 \frac{1}{2} \leq 0$? $[[4, 2\pi)]$

SFR-D2 A UPMMC drawer contains 10 Acad-Mem beads, 8 Sec-Spaf beads, and 11 Fin-Pub beads. Assuming that all beads of the same type are indistinguishable and the order of selection is irrelevant, determine the number of ways to select 16 beads if Victoria must select an even number of Acad-Mem beads, an odd number of Sec-Spaf beads, and a prime number of Fin-Pub beads. [14]

SFR-D3 Chords $\overline{TF}$ and $\overline{EI}$ of circle Γ intersect at a 60° angle at point X. The radius of Γ is 8 cm, $IX = 9$ cm, and $XE = 2$ cm. If $\overline{TX}$ and $\overline{XF}$ have integer lengths, find all possible values for the area of quadrilateral $TEFI$. $\left[\frac{99\sqrt{3}}{4}\text{ cm}^2, \frac{121\sqrt{3}}{4}\text{ cm}^2\right]$

Final Round

Easy

FR-E1 The probability $P(n)$ that a positive integer n is chosen is given by $P(n) = \frac{1}{2^n}$. Find the probability that a chosen number n is divisible by 3. $\left[\frac{1}{7}\right]$

FR-E2 Let S be the set of all natural numbers that can be expressed as $2^m 3^n$ where m and n are non-negative integers. What is the sum of the squares of the reciprocals of all the elements of the set S? $\left[\frac{3}{2}\right]$

FR-E3 Convert the decimal number 44 205 into hexadecimal (base-16) by setting the digits A = 10, B = 11, C = 12, D = 13, E = 14, and F = 15. $[\text{ACAD}_{16}]$

FR-E4 If $abc(a + b + c) = 4$ for all positive real numbers a, b, and c, what is the minimum value of $(a + b)(b + c)$? [4]

FR-E5 If $\log_{25} 10 = a$ and $\frac{\ln 16}{\ln 2 + \ln 5} = b$, express $\log 40$ as a fraction in terms of a and b. $\left[\frac{2 + 3ab}{4a}\right]$

Average

FR-A1 Mojo thinks that numbers expressed in positive number bases are too easy to handle. So, instead of writing in base 3, he writes in base −3. For example, we would write 15_{10} as 120_3. Mojo, however, would write this as 210_{-3}, since $15 = 2 \cdot (-3)^2 + 1 \cdot (-2)^1 + 0 \cdot (-3)^0$. Note that he also only uses non-negative numbers less than $|k|$ for the digits, where k is the base he's writing in. How would you write -10_{10} in Mojo's base -3 notation? [1212_{-3}]

FR-A2 A weird calculator operates in the following manner: it multiplies the input number by 3.4, removes the decimal part, and then subtracts 1 from the result. If this operation is performed thrice on a positive integer, the result is 444. Find the original number. [12]

FR-A3 During the MSA fair, you find a curious dice game. Upon paying 9 pesos, you can roll two special dice, both of which have one "1"-face, two "2"-faces and three "3"-faces. If the sum of the numbers are prime, you win 18 pesos (a net gain of 9 pesos). Otherwise, you get nothing. On average, how much do you expect to gain or lose in each roll? [Lose 0.50 pesos]

FR-A4 Careless Moses took the prime factorization of a number N and solved that the sum of its divisors is 1170. Unfortunately, while doing the prime factorization of N, he mistakenly wrote 2^3 instead of 2^5. What is the correct sum of the divisors of N? [4914]

FR-A5 Find the remainder when $7! \cdot (7^8+1)(7^4+1)(7^2+1)$ is divided by 17. [0]

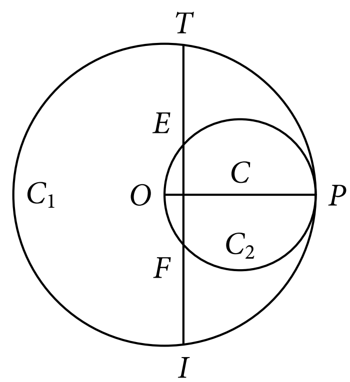
Difficult

FR-D1 If a polynomial $P(x)$ with integer coefficients satisfies $P(4) = 28\,800$ and $P(x^3) = (x^3+2x)(x^2-3x-4)P(x^2)$ for all values of x, determine $P(x)$. You may express your answer in factored form. [$P(x) = -x(x^2+8)(x+1)(x-64)$]

FR-D2 Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to the real number x and $\{x\}$ the decimal part of x. For example, $\lfloor \pi \rfloor = 3$ while $\{\pi\} = 0.14\,159\ldots$. Find all real values of x satisfying the equation $\frac{1}{2}\lfloor x \rfloor \{x\} = -20 + 2\lfloor x \rfloor$. $\left[x = 10, \frac{125}{11}, \frac{38}{3}, \frac{181}{13}\right]$

FR-D3 Given that $a_1 = 29$, $a_2 = 57$, and $a_n =$ last 2 digits of $(a_{n-1} + a_{n-2})$, for $n > 2$, find the remainder when $a_1^2 + a_2^2 + a_3^2 + \cdots + a_{2011}^2$ is divided by 10. [1]

FR-D4 Radius $\overline{OP}$ of circle C_1 with center O is the diameter of circle C_2. T and I are points on C_1 such that $\overline{TI} \perp \overline{OP}$ and C_2 trisects $\overline{TI}$ at points E and F. If $OP = 6$, find the distance of $\overline{TI}$ from the center of C_1. $\left[\frac{3}{4}\right]$



FR-D5 In the expansion of $(p - 2011pq + q^2)^k$, one of the terms is rp^3q^{k-3}, where r and k are positive integers. Find the value of $r + k$. [4]