*Codes:
E: for everyone
A: Algebra/Probability questions
G: Geometry/Trigonometry questions
N: Number Theory/Combinatorics questions

EASY

E1 If x and y are positive rational numbers, find $x+y$ if $x\lfloor x\rfloor = 153$ and $y\lfloor y\rfloor = 260$.

E2 Forty students took a final exam for which the passing score was 70. The mean score of those who passed was 75, the mean score of those who failed was 63 and the mean of all scores was 72. How many students did not pass the exam?

E3 Find the inverse function of $f(x) = \dfrac{(10^x+1)\left(10^{2x}+10^{-2x}+1\right)}{10^{-2x}+10^{-x}+1}$.

E4 Given $(x+y)^2 = 500$ and $(x-y)^2 = 400$, find x^3-y^3.

E5 Factor completely: $2x^2+3xy+9x+y^2+7y+10$.

E6 Ulyses asks Nadine to choose two numbers. He then instructs her to construct a sequence: "Add those two numbers then write the result after them. Add the second and third to get the fourth. Add the third and fourth to get the fifth and so on." Ulyses took a peek and saw that the seventh number was 39. He then tells Nadine the sum of the first 10 numbers she wrote. What was it?

A1 If the polynomial $f(x) = x^5+Tx^4+Ex^3+Fx^2+Ix-2520$ has 5 consecutive positive integral roots, find T.

A2 If x, y, z are three numbers chosen randomly from the set $\{-2011,-24,24,69,2011\}$, what is the probability that $x+yz$ is even?

A3 In a raffle for a waffle, five boys and five girls have entered, and only five winning tickets will be drawn. What is the probability that exactly two girls have won after five players have been picked?

G1 A city is shaped like a square with side $s = 1$ mile. A telephone company wants to build towers such that it can provide its services to the whole city. If each tower has a circular range with $r = \dfrac{1}{\pi}$ miles, at least how many towers does the telephone company need to build?

G2 Patty and Geo decided to make their meeting place secret. This place can be found $24\sqrt{3}$ meters from a certain tree that only the couple know. To reach the meeting place, one must walk 23 meters from this tree along the direction of 30° East of South, and afterwards, walk directly East. If Jayson found out which tree is the reference point, how many meters does he need to walk East to reach the meeting place?

G3 Three circles, each tangent to each other, have centers at the points M, I, and A. The triangle $\triangle MIA$ has side lengths 19 inches, 16 inches, and 11 inches. Find the sum of the areas of the three circles.

N1 Determine the number of trailing zeros at the end of 2011!.

N2 Find the greatest common divisor of 18456 and 26915.

N3 During an epic scene in an action movie, two opposing factions, each having five members, face each other. All ten of them load their respective guns and simultaneously fire at exactly one person from the other faction. How many possible ways could they have chosen who to shoot if all of them got shot (and subsequently died)?

AVERAGE

E1 Alvin has a special encoding technique. For every letter of a word, he finds the numerical value of the letter ($A = 1, B = 2, C = 3, \ldots, H = 8, I = 9, \ldots, Y = 25, Z = 0$) multiplies it by a number a, adds a number b, repeatedly adds or subtracts 26 until it becomes a number x where $0 \le x < 26$ and encodes it back to a letter. You recently discovered that Alvin encodes "HI" as "YO". How will he encode "TEFI"?

A1 Find all values of $x \in [0, 2\pi)$ at which $f(x) = \cot x \left(1 + 4\sin^4 x\right) + \tan x \left(1 + 4\cos^4 x\right)$ is minimized.

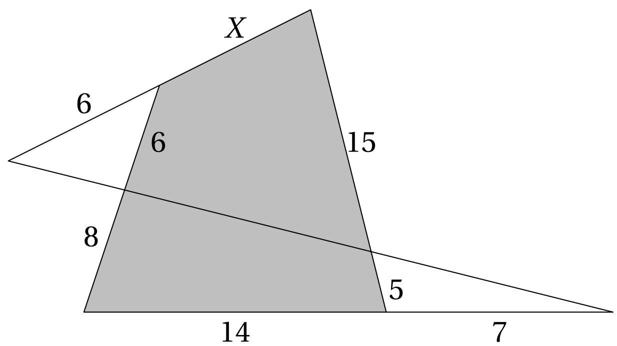
A2 Let A and B be the points of intersection of $x = 3y^2 - 2y - 6$ and the line $y = x$ and let P be a point on the parabolic arc AB. Find the maximum possible area of $\triangle ABP$.

A3 The function f from $\mathbb{R}$ to $\mathbb{R}$ satisfies the following equation for all $x, y \in \mathbb{R}$: $f\left(x + y\right) - f(x) = f(x) + f\left(3x - 2y\right) + y^2$. Find $f(3)$.

G1 Find the sum of the perimeters of all non-congruent triangles with integer side lengths having one side of length 1 cm and perimeter less than or equal 2011 cm.

G2 What is the largest possible perimeter of a rectangle inscribed in the region bounded by the graphs of $y = x^2 - 4$ and $y = 4 - x^2$?

G3 Stan the stairclimber was asked to scale a mountain. As he went up the mountain, he took measurements (in feet) as illustrated in the following figure. However, near the summit, he injured his leg so he cannot measure the length of the summit X. Help Stan determine the length of the summit.



N1 How many positive ordered triples (a, b, n) satisfy $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{n}{47}$ where $a \le b$?

N2 Determine the value of c if $\dfrac{7 + \sqrt{65}}{4}$ can be expressed as a continued fraction

$$\frac{7+\sqrt{65}}{4} = 2 + \cfrac{1}{0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{c + \cdots}}}},$$

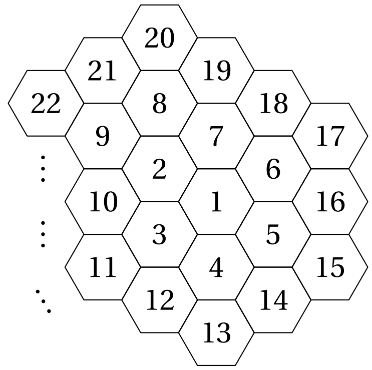
where the integers 2, 0, 1, 1, and c repeat (in that order) indefinitely.

N3 Suppose f is defined on all non-negative integers. We define $f(0) = 1$, $f(1) = 0$, $f(2n) = f(n) + 1$ and $f(2n + 1) = f(n)$ for all $n \in \mathbb{N}$. For how many integers $x \le 1000$ is $f(n) = 7$?

DIFFICULT

E1 Define the greatest integer function $y = \lfloor x \rfloor = n$ whenever $n \le x < n+1$. Determine all possible values of m such that the line $y = mx$ intersects $y = \lfloor x \rfloor$ exactly 2011 times.

E2 A honeycomb is shown in the figure.



What is the sum of the six numbers around the hexagon marked 2011?

A1 The numbers x and y are chosen between 0 and some real number s. Find s of the probability that the numbers 2, x and y form a triangle is $\frac{5}{8}$.

G1 Let $ABCDE$ be a regular pentagon with center O and side of length r. A star with another pentagon in the middle is formed by drawing the diagonals of $ABCDE$. Let $a = \sin 18°$. Find the length of each side of the smaller pentagon in terms of r and a.

N1 Two positive integers a and b are said to be *relatively prime* if they have no common positive factors except 1. Define $\varphi(n)$ to be the number of positive integers less than a positive integer n and are relatively prime to n. Find n such that $\frac{n}{\varphi(n)}$ is the maximum for $n \le 300$.