Questions not verbatim.

Credit goes to the following students, who managed to recall the questions (in alphabetical order). I hope I did not miss anyone.

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The $n$th problem is worth $n$ points and is allotted $n$ minutes.

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**1** Find all pairs of positive integers $(n, k)$ such that $(n+1)^k - 1 = n!$.

**2** Each square of a $69 \times 69$ board is colored either maroon or green. Each maroon square not adjacent to an edge is adjacent exactly 5 green squares out of its 8 neighbors. Each green square not adjacent to an edge is adjacent to exactly 4 maroon squares out of its 8 neighbors. Find the number of maroon squares.

**3** A ladder of 10 ft long is leaning against the wall and is touch a box with side length 2 ft. Determine the height from the top of the ladder to the floor. *(There are two possible answers; give the larger one.)*

**4** Find the sum of all monic quadratic polynomials $x^2 + ax + b$ with integer roots such that $1, a, b$ form an arithmetic progression.

**5** Find the longest side length of a triangle with altitudes 20, 28, and 35.

**6** Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{F_n^2 + 1}$, $F_n = 0$.

**7** Find the number of roots of $x^2 - \lfloor x^2 \rfloor = (x - \lfloor x \rfloor)^2$, $x \in [1, 30]$.

**8** Find all triples $(x, y, z)$ that satisfy $3(x-2)^2 + 8y - 2z = 30, 11x - \dfrac{8y}{3} - 2z = 30, 3x - 2y - 6z = -12$

**9** Let $A$ be the set of positive integers less than or equal to 2018. Let $B$ be positive integers not divisible by any number greater than 2 but less than 10. Find sum of elements of $A \cap B^{\mathrm{C}}$.

**10** 101 balls marked 1 to 101 are partitioned into two baskets A and B. Ball 40 is transferred form basket A to B. This increased the average of the numbers of the balls in each basket by $\dfrac{1}{4}$. Find the number of balls that are originally in basket A.

**11** The base of a square pyramid is $2018 \times 2018$ unit spheres. The next layer has $2017 \times 2017$ unit spheres, and so on. Find the height of the pyramid.

**12** Find the maximum value of $|x - y|$ if $x(7x-1) + y(7y-1) = 0$.

**13** Compute the exact value of $\cot\left(\dfrac{\pi}{7}\right) + \cot\left(\dfrac{2\pi}{7}\right) - \cot\left(\dfrac{3\pi}{7}\right)$.

**14** Find all triples $(x, y, z)$ of positive integers, $z$ being minimized, such that there exist positive integers $a, b, c, d$ such that $x^y = a^b = c^d$, $x > a > c$, $z = ab = cd$ and $x + y = a + b$.

**15** Evaluate $\displaystyle\sum_{k=1}^{\infty} \left( \frac{1}{24k+1} + \frac{1}{24k+11} - \frac{1}{24k-1} - \frac{1}{24k-11} \right)$.