



## 13<sup>th</sup> Philippine Mathematical Olympiad

National Stage, Written Phase

22 January 2011

*Time Allotment: 3 hours*

*Each item is worth 8 points.*

1. Find all nonempty finite sets  $X$  of real numbers with the following property:

$$x + |x| \in X \quad \text{for all } x \in X.$$

2. In  $\triangle ABC$ , let  $X$  and  $Y$  be the midpoints of  $AB$  and  $AC$ , respectively. On segment  $BC$ , there is a point  $D$ , different from its midpoint, such that  $\angle XDY = \angle BAC$ . Prove that  $AD$  is perpendicular to  $BC$ .

3. The 2011th prime number is 17483, and the next prime is 17489.

Does there exist a sequence of  $2011^{2011}$  consecutive positive integers that contains exactly 2011 prime numbers? Prove your answer.

4. Find all (if there is one) functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that satisfy the following functional equation:

$$f(f(x)) + xf(x) = 1 \quad \text{for all } x \in \mathbb{R}.$$

5. The *chromatic number of the (infinite) plane*, denoted by  $\chi$ , is the smallest number of colors with which we can color the points on the plane in such a way that no two points of the same color are one unit apart.

Prove that  $4 \leq \chi \leq 7$ .