EASY: 2 points each

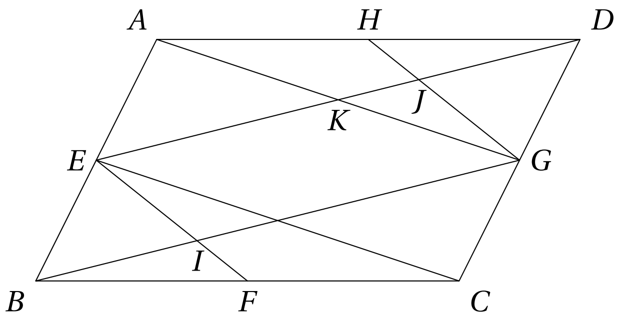
1 Find the minimum value of $1\circ 2\circ 3\circ 4\circ 5\circ 6\circ 7\circ 8\circ 9$ where each "$\circ$" is either a "$+$" or a "$\times$."

2 Compute the largest prime factor of $15! - 13!$.

3 Find all integer solutions of $2x^2 + 2xy + y^2 = 13$.

4 My name is Bruce Wayne. I graduated college at 15. After that, I spent $\frac{1}{4}$ of my working life training with *The League of Shadows* under Ra's al Ghul. I spent $\frac{1}{5}$ of my working life inside the Batcave, and I spent $\frac{1}{3}$ of my working life saving Gotham. For the last 13 years of my working life I've been with Selina Kyle. How old am I?

5 E, F, G, and H are the midpoints of segments $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{DA}$ respectively. Find the ratio of the area of the region $EIGK$ to the area of $ABCD$.



6 Find $4 + \cfrac{21}{4 + \cfrac{21}{4 + \cfrac{21}{4 + \cfrac{21}{4 + \cdots}}}}$.

7 Find all integral solutions of $a^2 = 25b^2 + 2012$.

8 In $\triangle ABC$, D and E are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. Let M be the centroid of $\triangle ABC$. The area of $\triangle DEM$ is 4. Find the area of $\triangle ABC$.

AVERAGE: 3 points each

1 In an arithmetic sequence, the 8th and the 20th terms are 54 and 114 respectively. Find the sum of the first 30 terms.

2 Solve for x in $\sqrt{2x+1} + \sqrt{x-3} = \sqrt{3x+4}$.

3 What is the smallest integer n for which any subset of $\{1, 2, 3, \ldots, 20\}$ of size n must contain two numbers that differ by 8?

4 The number $N = 700,245$ can be expressed as the product of three two-digit integers x, y, and z. What is $x + y + z$?

5 If $0 \le x, y \le 10$, what is the probability that the sum of x and y is between 2 and 8 inclusive?

DIFFICULT: 5 points each

1 Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$ for all positive integers m and n. Find a_{12}.

2 If $f\left(h\left(\frac{x-1}{x+1}\right)\right) = \frac{2x}{3x+1}$ and $f(x) = \frac{1}{x+1}$, what is $h(x)$?

3 Compute the number of ordered pairs of integers (b, c) with $-20 \le b, c \le 20$, such that the equations $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ share at least one root.

4 The centers of three identical spheres [that are tangent to one another] form an equilateral triangle. With the total surface area of the spheres being $60\sqrt{3}\pi$, what is the area of the triangle?

5 Define $S(n)$ as the last two digits of the sum of the factorials of all the divisors of n. For example, $S(2012)$ would be given by the last two digits of $1! + 2! + 4! + 503! + 1006! + 2012!$. Find the sum of all values $n < 100$ for which $S(n) = S(2012)$.

VERY DIFFICULT: 8 points each

1 Two circles P and Q have radii 20 and 12 respectively, and intersect at points X and Y. Extend $\overline{XY}$ to a point Z. Now, draw two lines from Z such that one intersects circle P at points D, E and the other intersects circle Q at points F, G. $\overleftrightarrow{EG}$ intersects circle P at H and circle Q at I. If $\angle ZEG = 90°$, and $EH = 24$, find the area of $DFGH$.

2 James Holmes, Andal Amapatuan, Sr., and Anders Behring Breivik were put in the same prison. The prison guards, who were fooling around assembled the three prisoners in a room. The guards gave each one a pistol and arranged a three-cornered pistol duel between the three killers. Of the three, James is the worst shot, hitting his target only 30% of the time. Amaptuan, Sr., a little better, is on-target 50% of the time, while Breivik never misses. The rules that the guards formed are simple: they are to fire at the targets of their choice in succession, and cyclically, in the order James, Ampatuan, Sr., Breivik, and so on until only one of them is left standing. (On each "turn," they get only one shot. If a combatant is hit, he no longer participates, either as a shooter or as a target.) Calculate the probability of James staying alive if he hits Breivik with his first shot.

3 A *vord* is a sequence of letters, where each letter can be "A," "B," or "C." The value of a vord with p A's, q B's and r C's is $p + 3q + 4r$; for example, AAAA is a vord with value 4 while ACAAB is a vord of value 10. How many vords have value 14 and do *not* end with "BC?"