A Semifinal A

Easy: 30s, 2pts

E1 A dartboard is made up of two circles with the same center. The smaller circle A has radius 6 and the bigger circle B has radius 10. Hitting the dart inside A will give you 5 points. Hitting the dart in the area outside A but within B will give you 3 points. What is the probability of getting 3 points in 1 throw, assuming the dart hits part of the board in the throw? $\left(\frac{16}{25}\right)$

E2 A person's core memories are formed in any order, with the exception that the core memory related to Family must be formed before the core memory related to Friendship. If someone has 7 unique core memories, 2 of which are Family and Friendship, how many ways could these memories be formed? (2520 ways)

E3 Let $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \cdots$. If $f\left(\frac{x}{x-1}\right) = 10$, find x. (10)

E4 Two clocks are currently displaying different times. The first clock is currently at 9:45 AM while the second clock is currently at 6:25 PM. If the first clock moves normally, while the second clock goes backwards, when will both clocks display the same time? (2:05 PM)

E5 If $x + \frac{1}{x} = 3$, find the value of $x^3 + \frac{1}{x^3}$. (18)

Average: 60s, 3pts

A1 David and Gabe are working on a group paper. However, since both of them are also busy doing other things, one usually starts writing the paper ahead of the other. If David starts writing the paper by himself and Gabe joins in 2 hours later, they can finish the paper in 5 hours. If Gabe starts writing the paper by himself and David joins in 2 hours later, they can finish the paper in 6 hours. How long will take to finish the paper if they start at the same time? (4.5 hours)

A2 If a unit sphere circumscribes a cube, what is the surface area of the said cube? (8 units^2)

A3 Let $a_1, a_2, a_3, \ldots, a_{17}$ be the roots of the equation $x^{17} + 17x - 2015 = 0$. Find the value of the sum $a_1^{17} + a_2^{17} + a_3^{17} + \cdots + a_{16}^{17} + a_{17}^{17}$. (34255)

A4 Square $PQRS$ is inscribed in a right triangle $\triangle ABC$ (with right angle at B) and RS is on AC. If $PB = 8$ and $PQ = 10$, then what is the area of $\triangle ABC$? (*fig) $\left(\frac{1369}{6}\text{ units}^2\right)$

A5 For what positive integers n is $\frac{20n + 874}{4n - 5}$ a positive integer? (9 and 226)

Difficult: 90s, 5pts

D1 Lu and David are having a problem solving contest. Being the older and more experienced solver, David already attempted 50 problems and got 49 problems correctly before Lu started 6 problems and answered 2 correctly. To make their competition fair, both of them will attempt the same number of problems from then on. What is the minimum number of questions Lu needs to solve correctly for which it is possible for him to get a higher ratio of correct answers to total questions solved than David? (13 questions)

D2 A pyramid has a base which is an equilateral triangle with side length 300 cm. The top of the pyramid is 100 cm above the center of the triangular base. Jerry the Mouse starts at a corner of the base of the pyramid and walks up the edge of the pyramid toward the vertex at the top. When Jerry has walked a distance of 138 cm, how many centimeters above the base of the pyramid is Jerry? (69 cm)

D3 Evaluate $\sum_{n=1}^{12} n^2 + \sum_{n=2}^{12} n^2 + \sum_{n=3}^{12} n^2 + \cdots + \sum_{n=1}^{12} n^2 + \sum_{n=12}^{12} n^2$. (6084)

D4 When a polynomial $P(x)$ is divided by $(x+3)$, the remainder is -5. When $P(x)$ is divided by $(x-5)$, the remainder is 11. What is the remainder when $P(x)$ is divided by $\left(x^2-2x-15\right)$? $(2x+1)$

D5 Let f be a function on all nonzero real numbers such that for all $x, y \in \mathbb{R}\backslash 0$, $f\left(4xy\right)+f\left(\frac{x}{y}\right) = \frac{4y^2+1}{4xy}$. Find $f(1)+f(2015)$. $\left(\frac{2016}{2015}\right)$

B Semifinal B

Easy: 30s, 2pts

E1 Find the value of $2015_6 + 2015_7$ in base 9. (1507_9)

E2 A circle with radius $\sqrt{\frac{6}{\pi}}$ has the same area as a regular hexagon. Find the length of each side of the hexagon. $\left(\frac{2\sqrt[4]{27}}{3} \text{ units}\right)$

E3 Jimmy has many bowties he can wear every day. He has 16 red, 12 yellow, 19 blue, and 11 black bowties. If he randomly takes one from his collection everyday and doesn't reuse bowties until all are used, what is the minimum number of days needed to guarantee he has worn at least 7 bowties of each color? (54 days)

E4 The remainder of 323 when divided by a number is 15. When 323 is doubled by the same number the remainder is 8. Find the value of the divisor. (22)

E5 Find the largest integer value of n such that 35! is divisible by 3^n. (15)

Average: 60s, 3pts

A1 Deany and Timoy are running a marathon. Timoy runs at 10 kph while Deany runs at 8 kph. After a certain amount of time, Timoy was ahead of Deany by 4 kilometers, but he got tired and started walking at 5 kph from then on. If Timoy and Deany started at the same time, after how many minutes will Deany overtake Timoy? (200 minutes)

A2 How many trailing zeroes are there in the base 4 representation of 2015!? (1002)

A3 A Fibonacci sequence $\{F_1, F_2, \cdots\}$ is a sequence such that the n term is the sum of the two previous terms for all integers $n > 2$. Suppose $F_1 = F_2 = 1$. If I get a random term from the first 2015 numbers in the sequence, what is the probability that it is odd? $\left(\frac{1344}{2015}\right)$

A4 There are 52 blankets in the cabinet. Twenty-seven are blue, 5 are red, and the remaining are white. In a dark morning, Jakov draws 3 blankets from the cabinet without replacement. Determine the probability of him getting 1 blue, 1 red, and 1 white blanket. $\left(\frac{27}{221}\right)$

A5 What is the measure of each angle of a regular polygon if it has 819 diagonals? $\left(\frac{1200^\circ}{7}\right)$

Difficult: 90s, 5pts

D1 Your AMS crush unexpectedly gave you his phone number but in your excitement you lost the valuable strip of paper. However, by some stroke of luck, you managed to remember the first four digits in the proper order: 0925. The remaining digits consist of 3, 6, 7, 8, 8, 8, 8 and one other number. You randomly guess a phone number based on the information you remembered, and call the number. What is the probability that your crush will be on the other end of the line? $\left(\frac{1}{1715}\right)$

D2 Suppose a given polyhedron has x faces, $\left(x^3+3x^2-2x+1\right)$ edges and $\left(2x^3-x^2-22x-11\right)$ vertices. Exactly how many faces must it have? (7 faces)

D3 When 29 is added to a number, it becomes a perfect square. When 100 is added to the same number, it becomes a perfect fourth power. What is the number? (1196)

D4 A polynomial of degree 4 and with a leading coefficient of 1 satisfies $f(1)=17$, $f(2)=34$, $f(3)=51$. Determine $f(0)=f(4)$. (92)

D5 Let p,q,r be prime numbers such that $p^2+3p+r=4q$, where $q>3$. Find all possible triples (p,q,r). (3,5,2)

C Final

Easy 30s, 2pts

Joy In flipping a fair coin exactly five times, what is the probability of getting three heads or three tails consecutively among the five flips? $\left(\frac{1}{2}\right)$

Sadness How many faces does a polyhedron with 1000 vertices and 2015 edges have? (1017 faces)

Anger What is the sum of the first 1000 odd numbers? (1 000 000)

Fear If a circle with radius 10 cm is circumscribed by an equilateral triangle, find the perimeter of the triangle. $\left(60\sqrt{3}\text{ cm}\right)$

Disgust If the squares of two consecutive odd numbers differ by 1000, what are the two odd numbers? (249 and 251)

Average 60s, 3pts

Joy With light traffic, it takes Timoy 20 minutes to get from Town A to Town B; meanwhile, in heavy traffic, it takes Timoy 60 minutes to travel the same distance. What is the latest Timoy can leave Town A, given that traffic will be heavy from 5:30 PM to 5:55 PM and that he has to be at Town B by 6:00 PM? Round off to the nearest minute. (5:23 PM)

Sadness How many trailing zeroes does 2015! have? (502)

Anger Let C and D be two circles with radii 3 cm, and with centers C and D, respectively. Denote by A and B the points of intersection of the two circles, E the second intersection of $\overline{CD}$ and circle D, and F the second intersection of $\overline{CD}$ and circle C. Given that $AB=3$ cm, find the length of $\overline{EF}$. $\left(\left(6-3\sqrt{3}\right)\text{ cm}\right)$

Fear If a die is to be rolled three times, what is the probability that the results obtained will be in strict ascending order? $\left(\frac{5}{54}\right)$

Disgust Two marbles are chosen (with replacement) from a box that has R red marbles and B blue marbles. If the probability that one marble of each color is drawn is equal to $\frac{15}{31}$, then what is the value of $\frac{R}{B}+\frac{B}{R}$? $\left(\frac{32}{15}\right)$

Difficult 90s, 5pts

Joy Your house is situated 6 kilometers north of the river and your girlfriend's house is located 8 kilometers east and 3 kilometers north of your house. If you have to pass by the river in order to check your appearance before your big date, what is the shortest distance (starting from your house) you can possibly travel to avoid unnecessary sweating? (17 km)

Sadness Given that a and b are both positive and $a > b$, find the ordered pair (a, b) that satisfies the system of equations

$$ab + a + b = 14$$
$$a^2 + b^2 + 2ab - 3a - 3b + 1 = 19$$

((4,2))

Anger Find the value of $\sqrt{(9900)(101)(102)+1}$. (10 099)

Fear Find the value of $\sum_{n=1}^{2014} \frac{7^{2015}}{49^n + 7^{2015}}$. (1007)

Disgust Janna and Claud play a game with a standard 52-card deck, with each player alternately drawing one card, with replacement. The first person to draw a red card wins. If Janna draws first, what is the probability that Claud wins? $\left(\frac{1}{3}\right)$

Very Difficult: 120s, 8pts

Joy Find the value of the infinite sum $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)(n+4)}$. $\left(\frac{1}{72}\right)$

Sadness A certain machine is powered by two AA batteries; it cannot function if one or both and defective. If you have 20 identical AA batteries—6 of which are defective—to be placed randomly in 10 of the machines, what is the probability exactly 6 of them will work? $\left(\frac{42}{323}\right)$

Anger In the game of chess, a new piece called the "super knight" is introduced, where it can make either one of the following movements:

1 Move 2 squares up, then 3 squares right.

2 Move 3 squares up, then 2 squares right.

If a super knight starts at the bottom left corner of a 22×25 chessboard, how many ways can it move to the top-right corner of the board? (84 ways)

Fear How many ordered triples (x, y, z) of positive integers are there such that $x + y + z = 23$ and x, y, z are all odd integers? (66 triples)

Disgust In a regular hexagon with side length 16 cm, another hexagon is made by connecting the midpoints of each of the sides. This goes on and on until infinity. Find the sum of the areas of all of the hexagons formed (including the original). $\left(1536\sqrt{3}\text{ cm}^2\right)$