Easy Round: 20 seconds each

E1 Compute the unique positive integer that, when squared, is equal to six more than five times itself. $[6]$

E2 David flips a fair coin five times. Compute the probability that the fourth coin flip is the first coin flip that lands heads. $\left[\frac{1}{16}\right]$

E3 The analog clock in front of you display the time 3:40. Compute the number of degrees in the smaller angle formed by the minute hand and the hour hand of the clock. $[130^\circ]$

E4 Isosceles trapezoid $ABCD$ has $AB = 10$, $CD = 20$, $BC = AD$, and an area of 180. Compute the length of BC. $[13]$

E5 The coordinates of three vertices of a parallelogram are $A(1,1)$, $B(2,4)$, $C(-5,1)$. Compute the area of the parallelogram. $[18]$

E6 An infinite geometric sequence has a first term of 12, and all terms in the sequence sum to 9. Compute the common ratio between consecutive terms of the geometric sequence. $\left[-\frac{1}{3}\right]$

Average Round: 40 seconds each

A1 James writes down three integers. Alex picks some two of those integers, takes their average, and adds the result to the third integer. If the possible final results Alex could get are 42, 13, 37, what are the three integers James originally chose? $[-20, 28, 38]$

A2 Consider a 4×4 grid of squares, each of which are originally colored red. Every minute, Piet can jump on one of the squares, changing its color and of its adjacent squares to blue. What is the minimum number of minutes needed for Piet to change the entire grid to blue? $[4]$

A3 For any positive integer $x \geq 2$, define $f(x)$ to be the product of the distinct prime factors of x. For example, $f(12) = 2 \times 3 = 6$. Compute the number of integers $2 \leq x \leq 100$ such that $f(x) < 10$. $[23]$

A4 Kelvin the Frog is trying to hop across the river. The river has 10 lily pads on it, and he must hop on them in a specific order (the order is unknown to Kelvin). If Kelvin hops to the wrong lily pad at any point, he will be thrown back to the wrong side of the river and will have to start over. Assuming Kelvin is intelligent and knows what he is doing, what is the minimum number of hops needed to reach the other side? $[176]$

A5 Let $ABCD$ be a convex quadrilateral whose diagonals AC and BD meet at P. Let the area of triangle APB be 24 and let the area of triangle CPD be 25. What is the minimum possible area of quadrilateral $ABCD$? $\left[49 + 20\sqrt{6}\right]$

A6 For how many triples (x, y, z) of integers between -10 and 10 inclusive do there exist reals a, b, c that satisfy $ab = x$, $ac = y$, and $bc = z$? $[4061]$

Difficult Round: 60 seconds each

D1 Find any quadruple of positive integers (a, b, c, d) satisfying $a^3 + b^4 + c^5 = d^{11}$ and $abc < 10^5$. $[(128, 32, 16, 4) \text{ or } (160, 16, 8, 4)]$

D2 A graph consists of 6 vertices. For each pair of vertices, a coin is flipped, and an edge connecting the two vertices is drawn if and only if the coin shows heads. Such a graph is *good* if, starting from any vertex V connected to at least one other vertex, it is possible to draw a path starting and ending in V that traverses each edge exactly once. What is the probability that the graph is good? $\left[\frac{507}{16\,384}\right]$

D3 A number n is *bad* if there exists some integer c for which $x \equiv c \pmod{n}$ has no integer solutions for x. Find the number of bad integers between 2 and 42 inclusive. $[25]$

D4 For how many pairs of nonzero integers (c, d) with $-2015 \leq c, d \leq 2016$ do the equations $cx = d$ and $dx = c$ both have an integer solution? $[8061]$

D5 Let ABC be a triangle that satisfies $AB = 13$, $BC = 14$, $AC = 15$, Given a point P in the plane, let P_A, P_B, P_C be the reflections of A, B, C respectively across P. Call P *good* if the circumcircle of $P_AP_BP_C$ intersects the circumcircle of ABC at exactly 1 point. The locus of good points P encloses a region $\mathcal{S}$. Find the area of $\mathcal{S}$. $\left[\frac{4225\pi}{64}\right]$

D6 Find the sum of all positive integers $n < 2015$ that can be expressed in the form $\left\lceil \frac{x}{2} \right\rceil + y + xy$, where x and y are positive integers. $[2\,029\,906]$

Tie-breaker Problems

TB1 Compute the smallest positive integer $x > 100$ such that every permutation of the digits of x is prime. $[113]$

TB2 We say that a number is *arithmetically sequenced* if the digits, in order, form an arithmetic sequence. Compute the number of 4-digit positive integers which are arithmetically sequenced. $[30]$

TB3 Let ABC be a triangle with $AB = 12$, $BC = 5$, $AC = 13$. Let D and E be the feet of the internal and external angle bisectors from B, respectively. (The external angle bisector from B bisects the angle between BC and the extension of AB.) Let ω be the circumcircle of $\triangle BDE$; extend AB so that it intersects ω again at F. Extend FC to meet ω again at X, and extend AX to meet ω again at G. Find FG. $\left[\frac{1560}{119}\right]$

DoD Alice and Bob are playing a game in which Alice has $\frac{1}{3}$ probability of winning, a $\frac{1}{2}$ probability of tying, and $\frac{1}{6}$ probability of losing. Given that Alice and Bob played a game which did not end in a tie, compute the probability that Alice won. $\left[\frac{2}{3}\right]$