1 *(90s)* Consider the following systems of linear equations: $1 + a + b + c = 1$, $27 + 9a + 3b + c = 5$, $125 + 25a + 5b + c = 9$. What is the value of abc? [3600]

2 *(30s)* One side of an isosceles triangle is 10 cm, while another is 12 cm. What is the absolute difference between the areas of the two possible triangles being described here? $\left[(5\sqrt{119} - 48)\,\text{cm}^2\right]$

3 *(60s)* A 50 m × 25 m × 2 m swimming pool has two drains: one on the floor that drains at 2 m^3/min, and one on the wall 0.8 m from the floor that drains at the same rate. To sanitize the pool and drainpipes, a high-speed pump fills the empty pool with cleaning liquid at a rate of 10 m^3/min, with the drains kept open throughout the entire process. The pump is closed when the pool is filled, and the liquid is drained out. How long will this entire process take? [1250 min]

4 *(30s)* What is the shortest distance between the point $A(12, 9)$ to any point of the circle $(x+4)^2+(y-3)^2 = 4$ if it has to touch the x-axis first? [18]

5 *(30s)* What is the least integer greater than M if $M = \log\frac{1}{2} + \log\frac{2}{3} + \cdots + \log\frac{2017}{2018} + \log\frac{2018}{2019}$? [−4]

6 *(60s)* There are 10 cards on a stack in front of you. Half of them are clubs, and the other half are diamonds. You draw a pair of cards five times. What is the probability that all resulting pairs contain exactly one club and one diamond? $\left[\frac{8}{63}\right]$

7 *(30s)* What is the remainder when $f(x) = (x+1)^6 + (x+2)^5 + (x+3)^4 + (x+4)^2 + (x+5)^1 + (x+6)$ is divided by $x + 4$? [701]

8 *(30s)* Find the sum of the coefficients of the function $f(x) = (x+1)(x-2)(x+3)(x-4)(x+5)(x-6)$? [−720]

9 *(60s)* Let $f(x)$ be a function that satisfies $(4 + x)f(x) - f(4 - x) = x^2 + 3x + 7$ for all real numbers x. Solve for $f(12)$. $\left[\frac{701}{65}\right]$

10 *(90s)* Find the positive solution to the equation $\dfrac{1}{x^2 - 10x - 29} + \dfrac{1}{x^2 - 10x - 45} - \dfrac{2}{x^2 - 10x - 69} = 0$. [13]

11 *(60s)* Find the exact value of $\left(\dfrac{\sin 20^\circ + \sin 100^\circ}{\cos 20^\circ + \cos 100^\circ}\right)^2$. [3]

12 *(30s)* How many ways can you arrange the letters in *MATHINEERS* such that the letters M, T, N, R always appear in that order? [75 600]

13 *(90s)* What is the remainder when $47^{23} + 23^{47}$ is divided by 1081? [70]

14 *(90s)* Let $3x$, $5y$, $7z$ form a geometric sequence while $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$ form an arithmetic sequence. What is the value of $\frac{x}{z} + \frac{z}{x}$? $\left[\frac{58}{21}\right]$

15 *(60s)* Let $f(x)$ be a real function such that 2019 is in the domain and range of f. Suppose that the equation $f(f(x)) \cdot (2019 + f(x)) = (f(x))^2$ holds for all x in the domain of f. What is the value of $f(2019)$? $\left[\frac{2019}{2}\right]$

16 *(90s)* Find the value of the infinite sum $\frac{1}{7} + \frac{3}{49} + \frac{5}{243} + \frac{7}{2401} + \cdots$. $\left[\frac{2}{9}\right]$

17 *(30s)* If $0 < \alpha, \beta < \dfrac{\pi}{2}$, and $\sin\alpha = \dfrac{5}{13}$ and $\tan\beta = \dfrac{3}{4}$, what is the value of $\cos^2(\alpha - \beta)$? $\left[\dfrac{3969}{4225}\right]$

18 *(DoD)* A chord AB of length 8 cm is drawn on a circle of radius 6 cm. A random point C is selected on the circle. What is the probability that the area of the resulting triangle ABC is at least 16 cm^2?

$$\left[1 - \frac{1}{\pi}\cos^{-1}\frac{\sqrt{5}-2}{3}\right]$$