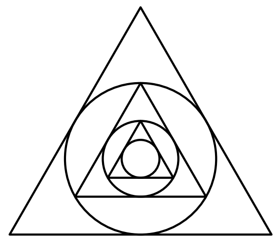
Survival Round I

Easy

SR1-E1 *(20s)* Issa always forgets her 4-digit ATM PIN. However, she knows that this number is the largest 4-digit number that is divisible by 6 and whose digits are in increasing order, starting from left to right. What must be her 4-digit PIN number? [4578]

SR1-E2 *(30s)* In the figure, the largest equilateral triangle has side x. The circle is then inscribed such that it is tangent to all sides of the triangle. Inside the circle is an equilateral triangle with its vertex on the circle. If this process continues infinitely, what is the sum of the perimeters of all the equilateral triangles? [$6x$ units]



SR1-E3 *(15s)* What is the least positive integer that becomes a perfect square when doubled and becomes a perfect cube when tripled? [72]

SR1-E4 *(15s)* Jason tells his someone's name on Wednesdays and Thursdays, tells a different name on Mondays, and tells either his someone's name or a different name on the other days. For six days in a row, Jeff observed Jason and obtained the following names told in order: May, Nadette, Lou, Desiree, Heart, Desiree. Determine the name of Jason's someone. [May]

SR1-E5 *(15s)* Betty bought 60 grams of 60% pure butter but it was too bitter. To make it taste better, she buys a bit of 100% pure butter and mixes it with the previous butter to produce a batter of 88% pure butter. How much butter did Betty buy to make the bitter butter better? [140 grams]

Average

SR1-A1 *(30s)* In basketball, a made field goal is worth either 2 or 3 points, while a made free throw is worth 1 point. In one game, Ramon scores a total of 36 points by making 5 out of 9 free throws and making 75% of all his shots, including free throws. How many 3-point shots did he make? [5]

SR1-A2 *(45s)* In an acute triangle $\triangle MSA$, $m\angle M = 60^\circ$ and $\tan M \tan S \tan A = 3$. Find $\tan M + \tan S + \tan A$. [3]

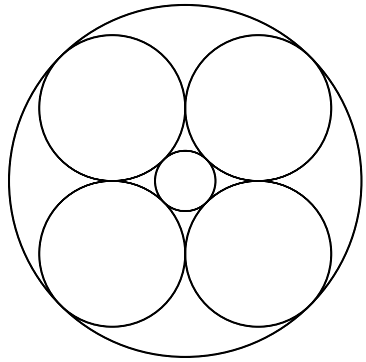
SR1-A3 *(30s)* Two numbers are chosen randomly from the interval $[0, 2010]$. What is the probability that the absolute value of their difference is greater than their mean? [$\frac{1}{3}$]

SR1-A4 *(45s)* Given two functions f and g, both defined from $\mathbb{R}$ to $\mathbb{R}$, such that $f(xg(y)) = 4xy$. If $f(23) \neq 0$, find $g(23f(23))$. [2116]

SR1-A5 *(45s)* How many 4-digit positive integers are there such that all of its digits are unique and the absolute value of the difference between the first digit and the last digit is 4? [616]

Difficult

SR1-D1 *(60s)* Four congruent circles are externally tangent to a larger circle and a smaller circle as shown in the figure. If the radius of the smallest circle is 2 cm, what is the area of the largest circle? [$6 + 4\sqrt{2}\,\text{cm}^2$]



SR1-D2 *(75s)* The positive integers a, b, c, d and e form a five-term geometric sequence with $a < b < c < d < e < 100$. What is the sum of all possible values of c? [129]

SR1-D3 *(90s)* How many numbers from 1 to 2011 have non-zero multiples that contain only the digits 0 and 1? [2011]

SR1-D4 *(120s)* The integers $36 + C$, $404 + C$ and $900 + C$ are all perfect squares, the square roots of which form an arithmetic sequence. Determine all possible values of C. [325]

Survival Round II

Easy

SR2-E1 *(15s)* Find the sum of all prime factors of 23!. [100]

SR2-E2 *(30s)* Find the constant term in the binomial expansion of $\left(2x - \dfrac{1}{x^3}\right)^5$. [80]

SR2-E3 *(30s)* Below is an excerpt from a wall post in Christine's social networking site account:

> **Christine J.** will gather courage and approach her crush 2010 minutes and 2010 seconds from now.
>
> November 26 at 7:53:30 am

At what time will Christine approach her crush? [5 : 57 PM]

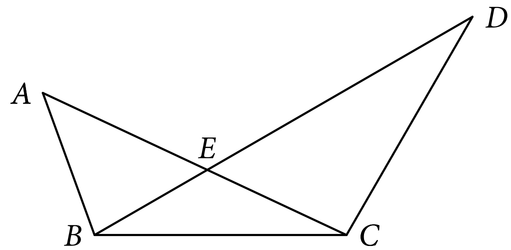
SR2-E4 *(25s)* Find all ordered pairs (x, y), $x, y \in \mathbb{Z}$ such that $x^2 - 3y^2 + 4 = 0$. [No solution]

SR2-E5 *(30s)* In a river flowing from south to north at a rate of 9 km per hour, an ugly duckling paddled slowly northwards while six sleek swans swam swiftly southwards. The magnitude of the rate of the paddling duckling, in km per hour, was equal to the square root of that of the swimming swans. If they met halfway from their starting points, how swiftly were the six sleek swans swimming? $\left[\dfrac{37 + \sqrt{73}}{2}\text{ km/hour}\right]$

Average

SR2-A1 *(30s)* What is the largest integer k such that $k^{2010} + 2k^{2009} + 3k^{2008} + \cdots + 2010k + 2011$ is divisible by $k + 1$? [1005]

SR2-A2 *(60s)* In the figure, $AB = AE$ and $CB = CD$. If $m\angle ECD = 70°$, what is $m\angle ABC$? [100°]



SR2-A3 *(45s)* Find a polynomial of minimum degree whose coefficients are integers such that $\sqrt{2}+\sqrt{6}$ is a zero of the polynomial. $[x^4 - 16x^2 + 16]$

SR2-A4 *(45s)* An n-tower is a pile of n right circular cylinders, all with height 1 unit, and with distinct integral radii from 1 to n. For integers k from 1 to $n-1$, the cylinder with radius k is glued to the cylinder with radius $k+1$ on their circular faces, with the smaller face completely attached to the bigger. Find a formula for the total surface area of the n-tower. [$(3n^2\pi + n\pi)$ square units]

Difficult

SR2-D1 *(60s)* In Jessy Leigh's planet, a day is equivalent to 20 hours, and an hour is still composed of 60 minutes. Jessy Leigh uses a 20-hour analog watch. Each full revolution of the minute hand still corresponds to one hour, and the hour hand moves in proportion to the movement of the minute hand. Find the first time between 7:00 and 8:00 when the minute and hour hands are perpendicular. [$7{:}06\frac{6}{19}$]

SR2-D2 *(90s)* Let j and p be integers such that $1 \le j < p$. For what values of j and p will the product $\frac{p-2j-1}{j+1}\binom{p}{j}$ be an integer? [All values of j and p]

SR2-D3 *(90s)* The circle constructed on side BC of $\triangle ABC$ such that BC is the diameter intersects sides AB and AC at D and E respectively. If $BC = 25$, $BD = 7$ and $CE = 15$, find the ratio of the areas of $\triangle ADE$ and $\triangle ABC$. [$\frac{9}{25}$]

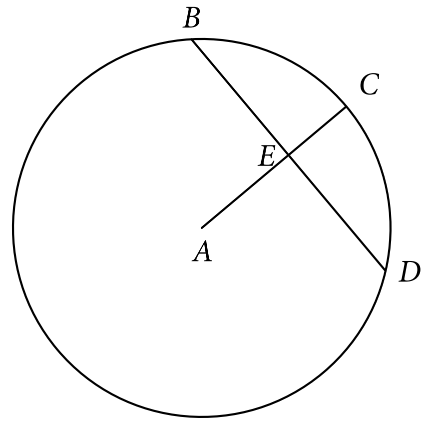
Quaterfinal Round

Easy

QFR-E1 *(30s)* How many real roots does the polynomial $x^{2010} - x^{2009} - x^{2008} - \cdots - x - 1$ have in the interval $(1,2)$? [1]

QFR-E2 *(25s)* Find the value of $a+b$ if $(a+\sqrt{a^2+1})(b+\sqrt{b^2+1}) = 1$. [0]

QFR-E3 *(20s)* In the figure, radius AC bisects chord BD at E. If $EC = 2$, and $BD = 8$, find the circumference of the circle. [10π units]



QFR-E4 *(30s)* Find the zeroes of the function f defined on $\mathbb{R}$ such that $f(x) = \begin{cases} x^3 - 1, & x \text{ is rational} \\ x^4 - x^2 - 2, & x \text{ is irrational} \end{cases}$.

$\left[1, \pm\sqrt{2}\right]$

QFR-E5 *(30s)* Unique digits are assigned to each of the letters in the set $\{A, B, G, H, I, M, N, R, T, Y\}$. What is the largest possible value for the difference $MATHIRANG - MATHIBAY$? [888 889 751]

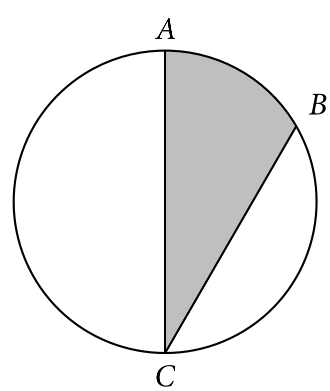
QFR-E6 *(30s)* May's mommy makes moist muffins only on Mondays. On the mth Monday of the year, she makes m muffins. In a certain year, she makes 1431 muffins. Find all possible dates in that year when May's mommy makes 10 moist muffins. [March 4 and March 5]

Average

QFR-A1 *(30s)* What is the square of the product of the last two digits in the sum of the factorials of the first 100 positive integers? [9]

QFR-A2 *(45s)* Find the number of possible ordered pairs of nonnegative integers (a, b), where $a, b \leq 50$ for which $3^a + 4^b$ is divisible by 5. [663]

QFR-A3 *(60s)* Given a circle with diameter AC and radius 3, if $m\angle ACB = 30^\circ$, what is the area of the shaded region? $\left[\left(\frac{3\pi}{2} + \frac{9\sqrt{3}}{4}\right) \text{ square units}\right]$



QFR-A4 *(45s)* Let $g(x) = \dfrac{1}{23^x + \sqrt{23}}$. Find the value of $g(-22) + g(-21) + \cdots + g(0) + \cdots + g(22) + g(23)$.

$\left[\sqrt{23}\right]$

QFR-A5 *(45s)* Given $m, s, a \in \mathbb{R}$. If $\dfrac{m^2+s^2+a^2}{sa+am+ms} = -2$, find the value of $\dfrac{m^2}{sa}+\dfrac{s^2}{am}+\dfrac{a^2}{ms}$. [3]

Difficult

QFR-D1 *(90s)* A solution of the equation $[x]\cdot\{x\} = x$, where $[x]$ denotes the greatest integer less than or equal to x and $\{x\} = x-[x]$ denotes the least nonnegative fractional part of x, is called an *MMC '06* number. If $a, b, c \neq 0$ are the three largest *MMC '06* numbers, find the value of abc. $[-\frac{3}{2}]$

QFR-D2 *(90s)* Convert the decimal number $1841\frac{13}{16}$ to base 2. $[(1\,110\,011\,001.1101)_2]$

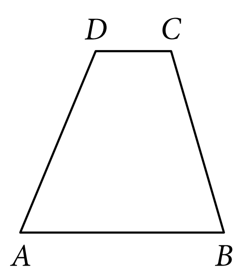
QFR-D3 *(75s)* Given $f(x) = \dfrac{1-\dfrac{1}{x^2+x+1}}{1-\dfrac{2x+1}{x^2+x+1}}$, where $x \neq -1, 0, 1$. If a sequence $\{a_n\}$ has a first term equal $f(x)$, and the succeeding terms have values equal to $f(a_{n-1}(x))$, find $a_{2011}(23)$. $[\frac{12}{11}]$

QFR-D4 *(90s)* Jessy Leigh the Higad is standing at the origin facing the positive y-axis and walks 128 units forward. She then turns 45° to the right and walks a distance equal to $\dfrac{\sqrt{2}}{2}$ of the previous step. She continues this process of turning 45° and walking forward a distance equal to $\dfrac{\sqrt{2}}{2}$ of the previous step indefinitely. What point on the Cartesian plane will she approach? $[(128, 128)]$

Semifinal Round

Easy

SFR-E1 *(20s)* The figure shows a trapezoid $ABCD$. Given that $AB = 27$, $BC = 25$, $CD = 10$ and $AD = 26$, find its area. [444 square units]



SFR-E2 *(20s)* Given $S_n = \dfrac{1}{1\times 3}+\dfrac{1}{3\times 5}+\dfrac{1}{5\times 7}+\cdots+\dfrac{1}{(2n-1)(2n+1)}$. Find S_{23}. $[\frac{23}{47}]$

SFR-E3 *(20s)* Ernie and Vicki decide to jog around the Academic Oval which has a circumference of 2.2 kilometers. Ernie jogs at a speed of 12 kilometers per hour while Vicki jogs at a speed of 10 kilometers per hour. They both start at noon in front of the Oblation statue, but jog in opposite directions. They continue jogging until they meet in front of the Oblation statue for the first time since noon. How many times did the two joggers meet around the Academic Oval? [11]

SFR-E4 *(25s)* Given $R = \log_4 25$ and $M = log 4$, express R in terms of M. $\left[\dfrac{2-M}{M}\right]$

SFR-E5 *(30s)* Tony and Tim, the thin twin tinsmiths, were working together in molding pieces of tin. Alone, Tim can mold 1836 pieces of tin in 3 hours while Tony, the thinner of the twin tinsmiths, can mold 1024 pieces of tin in 5 hours and 20 minutes. In how many minutes can the thin twin tinsmiths mold 2010 pieces of tin? [150]

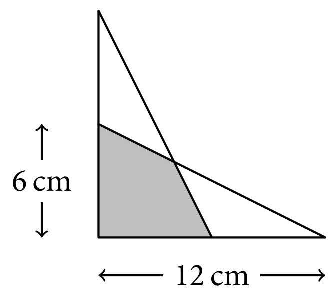
Average

SFR-A1 *(30s)* Let a sequence be defined by $x_{n+1} = 1 - \dfrac{2x_n}{1+x_n}$ for all $n \geq 1$. If $x_1 = 2100$, what is x_{2100}? $[-\frac{2099}{2101}]$

SFR-A2 *(45s)* Determine the sum of the series $\sum_{n=2}^{\infty} \ln\left(1 - \dfrac{1}{n^2}\right)$. $[-\ln 2]$

SFR-A3 *(30s)* Let A, E, F, J, M, N, O, R, and S represent distinct decimal digits. If $JASON + JEFF = RAMON$, find all possible values of $J + R$. [No solution]

SFR-A4 *(30s)* A card 12 cm long and 6 cm wide is cut along a diagonal to form two congruent triangles. Then the triangles are arranged as shown. Find the area of the region where the triangles overlap. [24 square units]



Difficult

SFR-D1 *(90s)* Find the sum of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \frac{1}{16} + \cdots$, whose terms are the reciprocals of positive integers which can be represented as a product of nonnegative integer powers of 2 and 3. [3]

SFR-D2 *(90s)* Determine the minimum value of $\dfrac{11}{\left|6 - 42\sin^2\dfrac{x}{2} - 20\sin x\right|}$. $[\frac{1}{4}]$

SFR-D3 *(120s)* Hana hates the letter R. When counting from 1, she skips numbers with digits 3 or 4. In Hana's count, there were 896 caterpillars in the tree. How many caterpillars were actually in the tree? [444]

Final Round

Easy

FR-E1 *(30s)* Evaluate $\sqrt{(5-\cos^2 15^\circ)^2} - \sqrt{(\sin^2 15^\circ - 3)^2}$. $\left[-\dfrac{\sqrt{3}}{2} + 2\right]$

FR-E2 *(30s)* On a regular 12-hour clock, determine the times between 5:00 AM and 6:00 AM when the minute and hour hands form an angle of 22°. [5:23$\frac{3}{11}$ am and 5:31$\frac{3}{11}$ AM]

FR-E3 *(30s)* Two fair six-sided dice are thrown simultaneously. What is the probability that the product of two face-up numbers is less than their sum? $[\frac{11}{36}]$

FR-E4 *(30S)* Find all possible values $R > 0$ such that $R = 2 + \cfrac{1}{0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{R}}}}$. $\left[\dfrac{3+\sqrt{21}}{2}\right]$

FR-E5 *(20s)* Peter Piper picked P pecks of pickled peppers, where P is the largest 2-digit prime numbber which can be expressed as a sum of P consecutive odd integers. How many pecks of pickled peppers did Peter Piper pick? [97]

Average

FR-A1 *(60s)* Find all real solutions of the equation $(\log x - 2)^3 = 7\log x - 20$. $[\frac{1}{10}, 1000, 10\,000]$

FR-A2 *(60s)* Let R be the smallest set containing all integers $m^5 - 5m^3 + 4m$, where m is a positive odd integer less than 100. A number is chosen at random from R. What is the probability that this number is divisible by 2010? $[\frac{2}{25}]$

FR-A3 *(45s)* How many ordered triples (x, y, z) of positive integers satisfy the equation $x + y + z = 15$ where x, y, z leave remainders of 0, 1, 2, respectively, when divided by 3? [10]

FR-A4 *(45s)* The degree measures of $\triangle MSA$ are prime numbers, with $m\angle M < m\angle S < m\angle A$. Find the smallest possible value of $m\angle A - m\angle S$. $[36^\circ]$

FR-A5 *(60s)* MG's favorite numbers are 2010-digit positive integers with 3 as the leftmost digit. In these integers, the number formed by taking any two consecutive digits must be divisible by 17 or 23. Find the sum of the units digits of MG's favorite numbers. [7]

Difficult

FR-D1 *(90s)* The letters C, M, P and U represent distinct decimal digits. The 2-digit numbers UP, PM, MM and MC are divisible by four consecutive odd prime numbers d, e, f, g, respectively, with $d < e < f < g$. Find the largest possible value of the 5-digit number $UPMMC$. [90 778]

FR-D2 *(120s)* Let $i = \sqrt{-1}$. Find the real part of the infinite sum $\left(\frac{1+i}{2}\right)^0 + \left(\frac{1+i}{2}\right)^1 + \left(\frac{1+i}{2}\right)^2 + \left(\frac{1+i}{2}\right)^3 + \cdots$. [1]

FR-D3 *(120s)* What is the remainder when $p(x) = x^{23} - 2x^{22} + 3x^{21} - \cdots + 23x - 24$ is divided by $x^2 - 1$? $[144x - 156]$

FR-D4 *(75s)* In convex pentagon $MIGEL$, $0^\circ < m\angle M \le m\angle I \le m\angle G \le m\angle E \le m\angle L$. The angle measures of the pentagon form an arithmetic sequence and two of its sides are parallel. Find all possible values for $m\angle M$. $[84^\circ]$

FR-D5 *(120s)* In her very thick MSA review book with pages numbered starting from 1, Leah writes her name on the pages whose numbers are divisible by 7. Counting from the last page, Jed draws a heart on every third page of the book. Leah now counts pages with both her name and a drawn heart. If Leah counts a total of 23 pages, find the maximum number of pages in the book without Leah's name or a drawn heart. [288]