Survival Round

Easy

SR-E1 *(20s)* Using his dial-up connection, Commander Goma downloads an artistic film at 20 kbps. After downloading 50% of the film, his download speed was reduced to a mere 10 kbps. What was his average download speed for the whole film? $\left[\frac{40}{3}\text{ kbps}\right]$

SR-E2 *(20s)* If $\frac{1}{729}$ of 81^{24} is 9^x, what is x? [45]

SR-E3 *(30s)* In the figure, $ABCD$ is a rectangle and $\overline{PD}$ is a diameter of the smaller circle tangent to the bigger circle. If the perimeter and area of $ABCD$ are 20 cm and 20 cm^2, respectively and the area of the smaller circle is 25π cm^2, find the distance between the centers of the two circles.* $[\sqrt{85}\text{ cm}]$

SR-E4 *(20s)* Let $\overline{abcdef}$ be a six-digit number such that $a < b < c < d < e < f$. How many such numbers are there? [84]

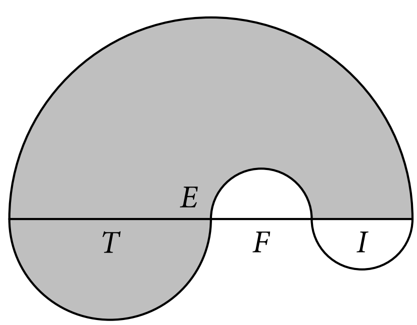
SR-E5 *(10s)* In how many ways can 10 people sit together in a round table if 3 people, Jayson, Krizza and Ina, want to sit together? Two seatings are considered the same if one is a rotation of the other. [30 240 ways]

SR-E6 *(15s)* Evaluate: $\cos 5° + \cos 10° + \cos 15° + \cdots + \cos 175° + \cos 180°$. [−1]

SR-E7 *(15s)* Every year that is exactly divisible by four is a leap year, except for years that are exactly divisible by 100; the centurial years that are exactly divisible by 400 are still leap years. For example, the year 1900 is not a leap year; the year 2000 is a leap year. How many leap years have passed since the introduction of the Gregorian calendar in 1582?† [105]

SR-E8 *(10s)* What is the largest number n such that the remainder when 749 993 and 250 007 are divided by n are equal? [499 986]

SR-E9 *(20s)* In the figure, points T, E, F and I are the midpoints of the diameters of the 4 semicircles as shown. $\overline{AB}$ is a segment containing the diameters of the four semicircles. If $EB = 6$ cm and $EF = IB$, what is the area of the shaded region? $\left[\frac{171}{8}\pi\text{ cm}^2\right]$



SR-E10 *(15s)* Ramon writes down his favorite function $f(x) = \frac{3x^2 + 5x - 6}{2x + 4}$. After seeing the equation, Jared computes for the inverse function and writes the function. Find the points of intersection of the graphs of the two functions. $[(-3, -3), (1, 1)]$

* The figure in the problem is impossible to construct. The following are additional details from the figure: Rectangle $ABCD$ is inscribed in a circle; A and D are the intersections of the two circles; P, A, and B are collinear.

† Note: This contest was held on December 2012.

Average

SR-A1 *(20s)* Evaluate: $(\cos^2 0° + \cos^2 1° + \cdots + \cos^2 90°)^2 + (\sin^2 0° + \sin^2 1° + \cdots + \sin^2 90°)^2$. $\left[\frac{8281}{2}\right]$

SR-A2 *(20s)* The sum of the terms of an infinite geometric sequence is 25 and the sum of the first two terms of the same sequence is 16. Find the common ratio of the sequence. $\left[\pm\frac{3}{5}\right]$

SR-A3 *(30s)* Find all solutions in real numbers to the system of equations $\ln 7xy = \ln(x^{\ln y})$, $\ln 7xz = \ln(x^{\ln z})$, $\ln yz = \ln(y^{\ln z})$. $\left[(x, y, z) = (7e^2, e^2, e^2) \text{ and } \left(\frac{1}{7}, 1, 1\right)\right]$

SR-A4 *(60s)* Point X_1 is placed on the side $\overline{X_0Z}$ of a triangle X_0Y_0Z such that $X_0X_1 = 45$ and $X_1Z = 90$. Furthermore, points $X_2, X_3, \ldots$ and $Y_1, Y_2, \ldots$ are drawn such that for any index i, the line $\overline{Y_iX_i}$ is parallel to $\overline{Y_0X_0}$ and the line $\overline{Y_iX_{i+1}}$ is parallel to $\overline{Y_0X_1}$. We let k be the smallest index such that $\overline{Y_kX_{k+1}}$ has a non-integral side length. If $X_1Y_0 = 108$, find the value of $KY_0X_1 + Y_1 + X_2 + \cdots + Y_{k-1}X_k$. $[260]$

SR-A5 *(30s)* Mathirang Mathibay questions can be classified into four different categories: Algebra, Geometry, Combinatorics, and Number Theory. Kwannieheart is asked to make five questions, each of which fall into exactly one of the above categories. One way he can choose the combination of categories is to make 4 Algebra questions and 1 Number Theory question. How many ways can he choose the combination of categories for the five questions? [56 ways]

SR-A6 *(30s)* Factor completely: $2x^2 + 3xy - 2y^2 - 10x - 15y + 8$. $[(2x - y - 8)(x + 2y - 1)]$

SR-A7 *(30s)* Determine the product of the coordinates of the vertex of the parabola whose roots are the squares of the x-intercepts of $f(x) = x^2 - 2x - 5$. $[-168]$

SR-A8 *(30s)* The sides of a quadrilateral $TEFI$ are 9, 9, 12 and 24 in no particular order. Two of its angles have equal sines but unequal cosines, yet this quadrilateral cannot be circumscribed by a circle. Find the quadrilateral $TEFI$'s area. $\left[\frac{1332}{5} \text{ units}^2\right]$

SR-A9 *(30s)* Rachelle wants to eat chocolates at an unusual pace. If she eats two chocolate bars today, she eats one on the next day, but eats three on the day after, after which she repeats the cycle and eats two again on the 4th day. Rachelle starts her weird chocolate diet on a Tuesday. On a certain day, Jessa sneakily eats 5 bars of chocolate. She did this only once, so she never got another again. Rachelle, not noticing that Jessa got some, still ate her usual chocolate quota. If Rachelle had 100 bars of chocolate at the start, on what day of the week did she eat her last chocolate bar and how many did she eat on that day? [Sunday, 2 bars]

SR-A10 *(30s)* When aliens from planet TEFI invaded Earth, TEFI time was introduced, where an earth-day is divided into ten TEFI hours, a TEFI hour into 100 TEFI minutes, and a TEFI minute into 100 TEFI seconds. Thus, instead of an earth-day starting at 0:00:00 and ending at 23:59:59, an earth-day now starts at 0:00:00 and ends at 9:99:99 in TEFI time. Rounded off to the nearest TEFI minute, convert the earth-time 20:12 to TEFI time. [8:42]

Difficult

SR-D1 *(45s)* Cedric decides to do his *Shockwave Dance* around a circular dance floor, of radius 1, centered at $(1, -1)$. If he decides to start from his favorite coordinate, $(1, -2)$, and travels in the following manner:

- He alternates from moving for 1 minute and pausing for 30 seconds.
- The first time he moves, his speed is 5° per minute.
- The second time, his speed is 10° per minute. The third minute, his speed is 15° per minute and so on.

Assuming his pauses occur for half a minute, what is the shortest time for Cedric to dance his way so that he may stop back to his favorite coordinate? [94 minutes]

SR-D2 *(45s)* The scores of "guilty", "not guilty", or "abstain" in the impeachment court may be represented by an ordered triple (G, N, A). The 23 senators vote any of the three aforementioned choices one by one in some order. A verdict is reached when at least $\frac{2}{3}$ of the senators have voted guilty or when such an event cannot be reached anymore. Upon reaching a verdict, the tally for the ordered triple (G, N, A) is declared the final score. How many possible "final scores" are there? [180]

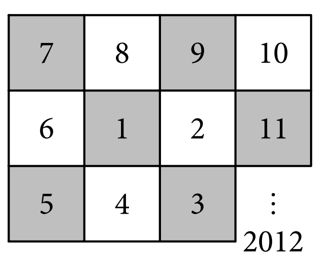
SR-D3 *(60s)* The sum of the squares of the roots of the polynomial $x^3 - 65x^2 + 1106x - 1960$ is 2013. What is the sum of the cubes of the roots of this polynomial? [64 835]

SR-D4 *(90s)* In a duel, Samurai Jack used a secret technique to slice Mojo Jojo's solid-hemispherical hat into an infinite number of pieces. The hat is cut with slices parallel to the base, at $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ of the way from the top. Little does Samurai Jack know that Mojo Jojo's mojo power is proportional to the total surface area of his hat. By how many percent does Mojo Jojo's mojo grow? $\left[111\frac{1}{9}\%\right]$

SR-D5 *(60s)* During Jolly Jayjam's one week Jamboree, Jayjam's jays jam some yams for Jayjam's Yam Jam at a rate where jays jam more yams each day, the number of which is double that of the previous day. On the first day, the jays jam a certain number x of yams for Jayjam's jam every hour, for y hours a day with each jar of yam jam needing z jams. At the end of the week, Jayjam's jays have jammed a total of 1524 yams. She reveals that on the 3rd day, it took her jays an hour to produce enough yams for $2\frac{2}{3}$ jars of Yam Jam. How many jars of Jayjam's Yam Jam did Jayjam's jays jam yam for during that week $x + y + z = 11$ and x, y, z are positive integers? [508 jars of Jayjam's Yam Jam]

SR-D6 *(75s)* We call a number an *n-repunit* if its base n representation consists only of 1's. For example, $111_5 = 31_{10}$ is a 5-repunit while $101_5 = 26_{10}$ is not. Let $n(b)$ denote the number of non-negative b-repunits less than 1000_{10}. Find $n(2) + n(3) + n(5) + n(7) + n(11)$. [26]

SR-D7 *(150s)* Find the sum of the diagonals of the figure, if it continues to 2012. [59796]



Quarterfinal Round

Easy

QFR-E1 *(15s)* A construction worker has 40 lbs of a mixture of cement and sand. When 5 lbs of the mixture is repalced by pure cement, the result is a mixture that is 30% cement. What percent of the original mixture was cement? [20%]

QFR-E2 *(20s)* How many distinct isosceles triangles are there with integer side lengths and perimeter 125? [31 triangles]

QFR-E3 *(20s)* Let I be on the face $ABCD$ of cube $ABCDEFGH$ with sidelength 25 cm such that AIB has area 500 cm^2. Find the volume of the figure $BIAHGFE$ as below.[‡] [$14\,062.5\ \text{cm}^3$]

QFR-E4 *(20s)* Find the solution set of $(y^2+4y)^2 - 2|y^2+4y| - 3 = 0$. [$\{-2 \pm \sqrt{7}, -1, -3\}$]

QFR-E5 *(15s)* Given: $\dfrac{\log 2}{\log 2 + \log 5} = 0.30$ and $\log 7 = 0.85$, find $\log_2 \sqrt{140}$. $\left[\dfrac{43}{12}\right]$

QFR-E6 *(20s)* The aardvark is a medium-sized, burrowing, nocturnal mammal native to Africa. It is commonly called as anteater. How many ways can you arrange the letters of the arrdvark such that there are no consecutive letter a's? [120]

Average

QFR-A1 *(30s)* A three-digit number is a *Tefi number* if its middle digit is greater than the sum of the first and last digits. How many Tefi numbers are there? [120 Tefi numbers]

QFR-A2 *(30s)* The sum of the first five terms of an arithmetic sequence is 5. If the product of the 1st and 2nd term is added to the product of the 4th and 5th term, the result is 326. Find the product of the five terms. [25 840]

QFR-A3 *(30s)* Find the coefficient of x^5 in the expansion of $(1+2x^{-1}+x)^8$. [168]

QFR-A4 *(30s)* Given that y is a multiple of 3 and $y \in [-99, 99]$, find the probability that y satisies the equation $7y^2 + 105y + 350 < 0$. $\left[\dfrac{2}{67}\right]$

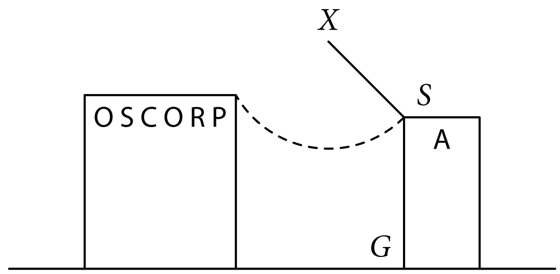
QFR-A5 *(30s)* Find the smallest number whose product of digits is $10! = 3\,628\,800$. [4 558 899]

Difficult

QFR-D1 *(45s)* Two players alternately roll a fair six-sided die. The player who first rolls a 3 wins the game. What is the probability that the first player wins? $\left[\dfrac{6}{11}\right]$

QFR-D2 *(80s)* In the figure, from the top of a building A at point $(1, -1)$, Spiderman creates a web that sticks to the origin, point X, and slings to the top of the Oscorp building to save Gwen Stacy. On the other hand, George Stacy (directly below Spiderman) decides to walk from the foot of building A to the foot of the Oscorp building. If Spiderman traveled a total distance of $\dfrac{7\sqrt{2}}{12}\pi$ units, how far will George need to travel? $\left[\dfrac{2+\sqrt{6}}{2}\text{ units}\right]$

[‡] This figure is impossible. The description of the figure is as follows: $ABCD$ is the top face of the cube; E, F, G, H are directly below D, C, B, A, respectively. The solid has the following visible edges: AI, AB, BI, AE, IE, IF, EF, BF, BG, and GF.



QFR-D3 *(120s)* Te and Fi are playing a game. Te is initially given a natural number $1 < n < 37$. Alternately, they divide the number they receive by a power of a prime that divides the number they received and passes it to the other player. Whoever passes the number 1 to his opponent wins. Assuming that each possible value for n has an equal chance of being given to Te and both players play using the optimal strategy, what is the probability that Te wins? $\left[\frac{24}{35}\right]$

QFR-D4 *(60s)* Let $M(18, 6)$ be such that line segment $\overline{MA}$ has a length of 24 units and passes through the point $S(15, 3)$. Find the equations of all lines parallel to the line $y = 3x$ and are 24 units away from point A. Write your answers in slope-intercept form. [$y = 3x - 108$ and $y = 3x + 36$]

Semifinal Round

Easy

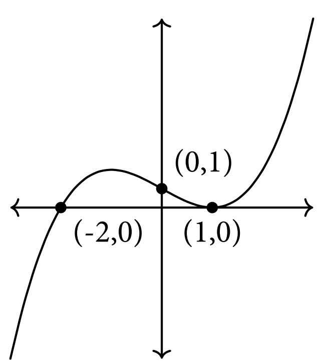
SFR-E1 *(20s)* Find the sum of all integers between 100 and 400 that end in 8. [7224]

SFR-E2 *(10s)* Shella sells seashells by the seashore. She can sell 100 seashells in 4 hours but her friend, Shiela, can do it faster. Shella already sold 25 shells when Shiela joined her. If 200 shells were sold $2\frac{1}{3}$ hours after Shiela joined Shella, how many shells can Shiela sell by the seashore in one hour? [50]

SFR-E3 *(15s)* The centers of circles M, S, and A lie along the diagonal of a square. Circle M and circle A are tangent to two sides of the square and circle S is tangent to both circle M and circle A. If each circle has a radius of 2 cm, fiind the area of the square. [$(48 + 32\sqrt{2})$ cm^2]

SFR-E4 *(10s)* If $f(x)$ and $g(x)$ are inverses of each other and it is given that $f(x) = \frac{3x + 11}{x - 3}$, determine $(g \circ f \circ g)(2)$. [-17]

SFR-E5 *(20s)* The graph of a cubic polynomial $y = ax^3 + bx^2 + cx + d$ is shown below. Find $ac - bd$. $\left[-\frac{3}{4}\right]$

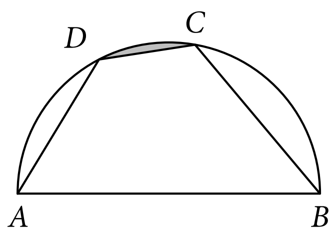


Average

SFR-A1 *(60s)* Let a, b, and c positive integers and $a < b < c$. If $abc + ab + bc + ac + a + b + c = 314$, find the ordered triple (a, b, c) that will minimize the value of $a^3 + b^2 + c$. $[(a, b, c) = (2, 4, 20)]$

SFR-A2 *(60s)* There are 6 houses, each with one mail box. If a mail man will distribute 6 letters, one addressed to each mail box, what is the probability that no letter is placed in the correct mail box? $\left[\frac{53}{144}\right]$

SFR-A3 *(45s)* Let $ABCD$ be inscribed in a semicircle with diameter $\overline{AB}$, measuring 12 inches. If $\angle BAD + \angle ABC = 108.75°$, find the area of circular segment CD. $\left[\left(30\pi - 18\sqrt{8 - 2\sqrt{6} + 2\sqrt{2}}\right) \text{in}^2\right]$



SFR-A4 *(30s)* Given $f(x) = 2x^2 + 2f(x-2)$, determine all possible functions $f(x)$. $[f(x) = -2x^2 + 16x + 80]$

Difficult

SFR-D1 *(90s)* Team Energy organizes a tennis tournament where 16 tennis players participate in a knock-out (single-elimination) tournament such that they are paired randomly at the start of each round. Each player is assigned a number from 1 to 16. The tournament is rigged in such a way that a player numbered a wins over a player numbered b if $a > b$. What is the probability that player 10 meets player 16 at the finals? $\left[\frac{1}{5720}\right]$

SFR-D2 *(90s)* Geoff the Geometer is drawing a right triangle with legs of length 5 cm and 12 cm. Like any good student of geometry, Geoff marks the right angle with a box. Geoff decides to play around with his figure and discovers that if the box has side 1 cm, he can draw a circle tangent to the 12 cm leg, to the hypotenuse and to the corner of the box which is in the interior of the triangle. Help Geoff find the radius of the circle. $\left[\frac{56 - \sqrt{86}}{25} \text{ cm}\right]$

SFR-D3 *(90s)* S is a set where its elements are $f(x)$, such that x is a positive integer from 1 to 2012. The function $f(x)$ is defined as $f(x)$ = the smaller permutation of the digits of x, e.g., $f(2012) = 122$, $f(432) = 234$. (Note that we consider $0122 = 122$.) How many elements does S have? $[588]$

Final Round

Easy

FR-E1 *(20s)* The perimeter of an equiangular octagon is equal to the diagonal of the square. The sides of the octagon alternate in length. Each side with length M is next to a side S is $1 : \sqrt{2}$ and the area of the octagon is 63 cm^2, find A, the area of the square. $\left[\left(216 + 144\sqrt{2}\right) \text{cm}^2\right]$

FR-E2 *(20s)* A circle is tangent to the line $x - y = 2$ at $(4, 2)$. If the center of the circle is on the x-axis, find the area of the circle. $[8\pi \text{ units}^2]$

FR-E3 *(20s)* Given that p and q are prime numbers, and m and n are positive integers such that $p = mn$ and $q = 3n + m$. Find all possible values of $\dfrac{p^q + q^m}{m^n + n^p}$. $[\{19, 35\}]$

FR-E4 *(20s)* Kebong the Rice Equilateral-Triangle Prism Vendor has just run out of tasty *nori* seaweed to put on his prisms, and so, he makes rice spheres instead, with diameter D, equal to the standing height of a rice equilateral-triangular prism, which to him, "looks much more". If he knows that the thickness of a prism is 3 cm, find D so that he can cheat his customers and sell the same amount of rice without the seaweed. $\left[\dfrac{6\sqrt{3}}{\pi}\text{ cm}\right]$

FR-E5 *(10s)* Determine the solution set of the equation $x^{\log x} = 100x$. $[\{100, 0.1\}]$

Average

FR-A1 *(30s)* Let $f(x) = 100x - 990\left\lfloor \dfrac{x}{10} \right\rfloor - 99\left\lfloor \dfrac{x}{100} \right\rfloor$, where $\lfloor a \rfloor$ denotes the greatest integer less than or equal to a. How many positive integers x less than 1000 satisfy $f(x) = x$? $[99]$

FR-A2 *(45s)* Determine all solutions of the equation $\cos(7\theta) + 2\sin(2\theta)\sin(5\theta) + 1 = 0$ where $\theta \in [0°, 360°)$. $[\theta \in \{60°, 180°, 300°\}]$

FR-A3 *(20s)* Evaluate $\left\lfloor \dfrac{2012^3}{2008 \cdot 2010} - \dfrac{2008^3}{2010 \cdot 2012} \right\rfloor$. $[16]$

FR-A4 *(60s)* Let A be a set where every element $s \in A$ satisfies the following:

(a) s is a multiple of 21.

(b) $0 < s < 2012$

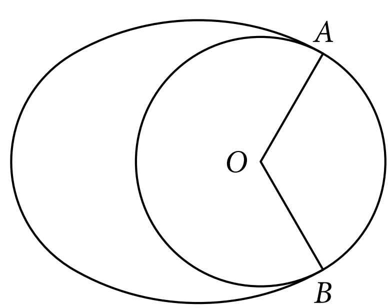
(c) s is 2 greater than a multiple of 19.

Find the sum of the elements of A. $[4095]$

FR-A5 *(45s)* Let p be a prime number. We define n to be the p-adic value of a positive integer x i.e. $n = p\text{-adic}(x)$, if n is the largest nonnegative integer such that p^n divides $x!$. Find the minimum x such that $2\text{-adic}(x) = 5\text{-adic}(2012)$. $[508]$

Difficult

FR-D1 *(120s)* In technical drawing, the *oval* is a sort of "ellipse" which can be drawn using a straightedge and a compass. It consists of two pairs of opposite and congruent circular arcs, with the tangent lines at the endpoints of any two adjacent arcs coinciding with each other. In the figure below, the arc of the small circle belonging to the oval has endpoints A and B.



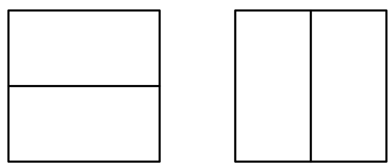
If arc AB measures 120°, $OA = 6$ cm, and the greatest distance between any two points on the oval is 18 cm, find the area of the oval. $\left[(72\pi - 18\sqrt{3})\ \text{cm}^2\right]$

FR-D2 *(105s)* After learning about sequences in MSA, Anjo constructs a sequence of eleven numbers in the following manner:

- His sequence starts with the number 1.
- The next number in the sequence is either one more than the previous number or one less than the previous number.
- His sequence will not contain a nonpositive number (i.e. from 1, he will neer make the next number 0).
- His sequence ends with the number 1.

How many such sequences can he make? [42]

FR-D3 *(75s)* There is one way to tile a 2×1 rectangle with a 2×1 domino, and as shown in the figure, there are two ways to tile a 2×2 square with 2×1 dominoes. Additionally, there are 754 011 380 476 346 429 ways to tile a 2×91 rectangle. How many ways can a 2×12 rectangle be tiled using 2×1 dominoes? [233]



FR-D4 *(105s)* Let $f(x) = x^2 + 2x + a$ and $g(x) = x^2 - 2x + a$, for some $a \in \mathbb{R}$. For what values of a does $(g \circ f)(x)$ have 3 distinct real roots? [no such values exist]

FR-D5 *(120s)* In the figure below, $ABIE$ is a rectangle, B is on the circle centered at O, and $\overline{EI}$ is tangent to the circle at N. If $PE = 13$ in, $IS = 5$ in, and $ON = 65$ in, find the sum o the lengths of the segments contained in the interior of the circle. [274 in]