Easy

General

E1 Evaluate: $\frac{3}{(2)(6)} + \frac{3}{(3)(7)} + \frac{3}{(4)(8)} + \cdots$. $\left[\frac{77}{80}\right]$

E2 What is the sum of all values of x that will satisfy the equation $\log_2 x \cdot \log_5 x \cdot \log_{11} x = \log_2 x \cdot \log_5 x + \log_2 x \cdot \log_{11} x + \log_5 x \cdot \log_{11} x$. $[111]$

E3 Let $x_1, x_2, x_3, x_4, \ldots, x_{10}$, and x be real positive numbers such that $y < x_1 < 2y, 3y < x_2 < 4y, 5y < x_3 < 6y, \ldots, 19y < x_{10} < 20y$. What is the value of y of $\sum_{i=1}^{10} (|x_i - (2i-1)y| + |x_i - (2i)y|) = 2013$? $\left[\frac{2013}{10}\right]$

E4 Find the sum $\frac{1}{2\sqrt{1}+1\sqrt{2}} + \frac{1}{3\sqrt{2}+2\sqrt{3}} + \frac{1}{4\sqrt{3}+3\sqrt{4}} + \cdots + \frac{1}{2013\sqrt{2012}+2012\sqrt{2013}}$. $\left[1 - \frac{\sqrt{2013}}{2013}\right]$

E5 In a race, at the nth station, a person has to toss a fair die n times. If the sum of the points of these n tosses is bigger than 2^n, the person is said to have crossed the station. At most how many stations can a person cross? $[4]$

E6 Rency and Jengy are hired to paint a long fence. If Rency works by himself, he could paint the fence in 4 hours. If Jengy works by himself, he could paint the fence in 3 hours. Rency starts painting the fence from one end, and Jengy begins painting the fence from the other end one hour later. They both work until the fence is fully painted. How many hours did Rency work? $\left[\frac{16}{7}\right]$

Algebra and Probability

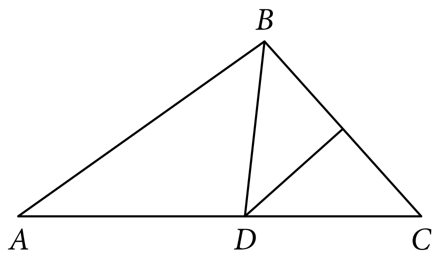
A1 In video games, hit rate is determined by a system called the *Random Number Generator* (RNG). Every time it is needed, the game rolls a number from 0 to 99, with each number having an equal probability of being generated. If the displayed hit rate were x%, and the number generated were less than x, then the attack would land. In this case, we have a perfect representation of the actual chance of hitting an enemy. But most games use the 2-RN system, where the game rolls two RN's and averages it. If the average is lower than the displayed hit rate, the attack would land. For a hit rate of 1%, the true hit would be 0.03% and for a hit rate of 90%, the hit rate would be 98.10%. What then is the true hit value if the displayed hit rate is 60%? [68.40%]

A2 A single shuttle travels from People 360° main branch to its Carmona branch and vice versa starting at 7:00 AM. A trip takes exactly 30 minutes without any delays. If an employee plans to arrive at the main branch sometime between 8:30 AM and 10:00 AM inclusive, what is the probability that he will wait for more than 25 minutes before the shuttle leaves, if there had been a delay for 5 minutes in all trips following the first one? $\left[\frac{13}{18}\right]$

A3 Victoria of Repahpeepz 08A forgot the distribution property of multiplication over addition while solving an equation but still got the correct answer. For how many ordered triples (x, y, z), each of which are between 0 and 9 inclusive, are there such that $x \times (y + z) = xy + z$? $[190]$

Geometry and Trigonometry

G1 In $\triangle ABC$, side AC and the perpendicular bisector of BC meet at point D, and BD bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, find the area of $\triangle ABD$. $\left[14\sqrt{5}\text{ units}^2\right]$



G2 If $\cos x + \sin x = \frac{1}{2}$, find the exact numerical value of $\cos^6 x + \sin^6 x + 2\sin^3 x \cos^3 x$. $\left[\frac{121}{256}\right]$

G3 Two circles of radius one and a circle of diameter one are drawn on a plane so that each of them is touching the others at one point. Find the radius of the largest circle that is tangent to all three of the circles in terms of the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$. $\left[\frac{1}{2} + \varphi\right]$

Number Theory and Combinatorics

N1 Four numbers P, T, B, A satisfy $P < T < B < A$. Each of the six possible pairs of distinct numbers has a different sum. The four smallest sums are 1, 2, 3, and 4. What is the sum of all possible values of A? $\left[\frac{15}{2}\right]$

N2 A pack of cards can be divided into two or more piles and piled in such a way that each pile would have one card more than the previous pile. For example, there is only one way a pack of 52 cards can be divided as such: 8 piles, with the largest pile having 10 cards and the smallest pile having 3 $(3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 52)$. If Kebong the Kardmaster had enough packs of 63 cards equal to the number of ways 63 cards can be arranged as such, how many piles would he create? [27]

N3 How many nonpositive integers N are there such that $N^4 + 4$ is prime? [1]

Average

General

E1 Dom and Jasmine have to move 16 small boxes and 10 large boxes. The table indicates the time that each person takes to move each type of box. If they start moving the boxes at 1:00 PM, what is the earliest time at which they can finish moving all the boxes? [1:43 PM]

	Dom	Jasmine
Small box	3 minutes	2 minutes
Large box	5 minutes	6 minutes

Algebra and Probability

A1 Let α be a nonzero complex number(s) of z will $|f_\alpha(z)| = 1$ if $f_\alpha(z) = \frac{z^2\alpha^2 + 2z\alpha + 1}{\alpha^2 + 2\overline{z}\alpha + \overline{z}^2}$ where $\overline{z}$ is the conjugate of z? $[\{z \in \mathbb{C} \mid |z| = 1\}]$

A2 Seventy-four people are lined-up to board a sleeper-tram. Each has a ticket with an assigned bunk bed. However, Kevin, the 26th person in line, lost his ticket and randomly takes a bed. After that, the next person takes his assigned bed if it is unoccupied, and takes an unoccupied bed at random if his assigned bed is occupied already. What is the probability that Christopher, the 73rd person in line gets to occupy his assigned bed? $\left[\frac{1}{2}\right]$

A3 Ramon and Daniel are playing a certain game. Daniel thinks of a certain fraction then Ramon rolls a die five times. For each die roll, if the number that appeared in the die is less than or equal to 4, Daniel multiplies his fraction by $\frac{4}{7}$. If the number on the die is greater than 4, Daniel multiplies his current fraction by $\frac{2}{5}$. If Daniel starts with the fraction $\frac{7}{8}$, what is the probability that his number will become $\frac{32}{1225}$ after the game? $\left[\frac{80}{243}\right]$

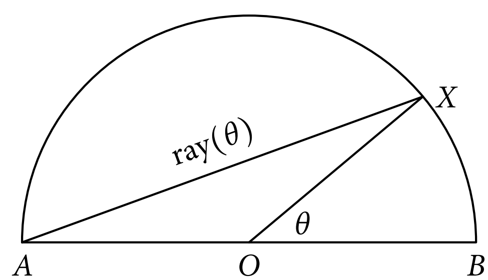
Geometry and Trigonometry

G1 Let S and A be the intersections of the graphs of $3x + 4y = 12$ and $9x^2 + 16y^2 = 144$. How many such real points M, satisfying $9x^2 + 16y^2 = 144$, are there such that the area of $\triangle MSA = 3$? $[2]$

G2 Batista tried to define circular functions of his own, and his favorite one (shown below) he calls $\text{ray}(\theta)$. Given any central angle θ with initial side $\overline{OB}$ and terminal side $\overline{OX}$ on a unit circle with $\overline{AB}$ as diameter, he defines $\text{ray}(\theta)$ to be the length of AX. If x is the smallest positive angle such that

$$\text{ray}(x)\,\text{ray}(2x)\,\text{ray}(4x)\cdots\text{ray}(2^{2013}x) = 1,$$

how many zeroes does the base 2 representation of $\frac{2\pi}{x}$ have? $[2012]$



G3 Determine the side length of a regular tetrahedron inscribed in a unit sphere. $\left[\frac{2\sqrt{6}}{3}\right]$

Number Theory and Combinatorics

N1 In how many ways can w, x, y, and z be chosen from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so that $w < x < y < z$ and $w + x + y + z$ is a multiple of three? $[72]$

N2 Splitting a number n means expressing it as a sum of consecutive positive integers. Moses can split 204 into four consecutive positive integer addends, that is, $502 + 503 + 504 + 505$. Note that the largest addend in this split is 505. He can actually split 2014 in different ways. If he splits 2014 in such a way that the split has the most number of addends, what is the largest addend in this split? $[64]$

N3 For what positive values of k will $\sqrt{k^2 - (2+p)k + (p+1)}$, where p is an odd prime, be an integer? $\left[\frac{(p+1)^2}{4} + 1\right]$

Difficult

General

E1 Janina was able to "extend" the complex number $\mathbb{C}$ into what she calls the *Kompurekkusu* numbers, $\mathbb{K}$, by including her favorite letter, j. This j behaves much like i except that $j^2 = 1$ but j is not any complex number (particularly not 1 nor −1). For $r, s, t, u \in \mathbb{R}$ how many solutions $\kappa = (r + si) + (t + ui)j \in \mathbb{K}$ are there such that $\kappa^4 = 4$ do i and j both appear with nonzero coefficients? [10]

E2 For $0 < G < \sqrt{2}H$, let D be the largest number such that some $D \times G$ rectangle is contained in a square of edge length H. Determine the value of D in terms of G and H.

$$\left[D = \begin{cases} H, & \text{if } G \in [H\sqrt{2} - H, H] \\ H\sqrt{2} - G, & \text{otherwise} \end{cases} \right]$$

Algebra and Probability

A1 Bogart wishes to be the very best Jejémon card collector by collecting all 20 Jejémon cards. Unfortunately, the only way to get them is by buying a sealed packet containing a random Jejémon card inside. If Bogart already has 17 (distinct) Jejémon cards, what is the expected number of packets he needs to buy before he can complete his collection?

Note that the expected number of cards to obtain a new Jejémon card is given by the formula $\sum_{k=1}^{\infty} k \cdot p(k)$ where $p(k)$ is the probability that Bogart gets a new Jejémon card after buying exactly k packets.

$$\left[\frac{110}{3} \right]$$

Geometry and Trigonometry

G1 In the figure, $ABCD$ is an isosceles trapezoid, and F is the intersection of its diagonals. If $AF : FC = 3 : 7$, $AB : BC = 26 : 37$, and $EJ \parallel AB \parallel CD$, find the ratio of the area of the shaded region to that of $ABCD$.*

$$\left[\frac{1}{40} \right]$$

Number Theory and Combinatorics

N1 Matthew Ramathibay XXVI has been tasked completely cover a rectangular room with special rectangular mats whose dimensions are 2×1. When k is a non-negative integer, we define a_k to be the number of ways he can cover a room with dimensions $3 \times k$ using the mats. For example, $a_6 = 41$. Let m_k be the remainder when a_k is divided by 7. How many positive integers of k less than 2013 are there such that $m_k = 3$? [252]

* The figure is an impossible construction. A description is as follows: E and J are points on AD and BC, respectively, such that F is on EJ. G, H, I are the respective intersections of JD and AC, JD and EC, and EC and BD.