Survival Round

Easy

SR-E1 *(20s)* The 16 members of Repahpeepz 08A want to adopt chinchilias as pets. The total number of chinchillas they would be adopting is same as the number of ways you can arrange the letters they would be adopting is same as the number of ways you can arrange the letters of the word *CHINCHILLA* such that the first letter and the last letter are not the same, and they agreed that they would not be naming an equal number of chinchillas. As a member of the group, how many names should Vic come up with? [12 915]

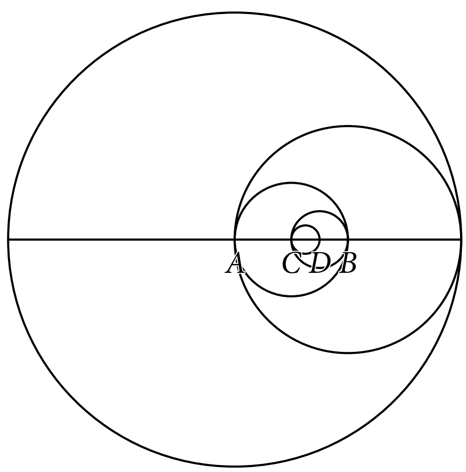
SR-E2 *(20s)* How much would it take to completely surround a ₱5 coin with ₱5 coins in ten layers such that each coin is tangent to at least three coins? [₱1650]

SR-E3 *(15s)* MSA creates a computer program that can solve for the roots of the polynomial $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ if the set of its ordered coefficients, in this case $(a_n, a_{n-1}, \ldots, a_1, a_0)$, is given as input. If $y = P_n(x)$ intersects $y = x$, what input generates a polynomial whose zeros are the intersection? $[(a_n, a_{n-1}, \ldots, a_1 - 1, a_0)]$

SR-E4 *(20s)* The greatest common divisor and the least common multiple of two positive integers are 12 and 168, respectively. If their sum is 108, what is the sum of their squares? [7632]

SR-E5 *(25s)* An integer n is randomly chosen from 1 to 48. What is the probability that the chosen integer will make the expression $\dfrac{n}{50-n}$ a perfect square integer? $\left[\dfrac{1}{16}\right]$

SR-E6 *(30s)* Given a circle of radius r, a *crescent* is formed by removing a smaller circle of radius $\dfrac{r}{2}$ whose center is the midpoint of the radius of the bigger circle. Crescent A's opening is facing east. In the figure below, the circle goes on indefinitely; find the total area of crescents whose opening is facing east, if circle A has radius 1. $\left[\dfrac{4\pi}{5}\text{ units}^2\right]$



SR-E7 *(20s)* Find $\sin 90^\circ \cos 80^\circ \sin 70^\circ \cos 60^\circ \sin 50^\circ \cos 40^\circ \sin 30^\circ \cos 20^\circ \sin 10^\circ \cos 0^\circ$. $\left[\dfrac{1}{256}\right]$

SR-E8 *(15s)* What is the sum of all postitive even factors of 900? [2418]

SR-E9 *(25s)* Aaron and Pat share a piece of land. The ratio of the area of Aaron's portion to the area of Pat's portion is 3 : 2. The entire piece of land is covered by carroots and corns in the ratio 7 : 3. On

Aaron's portion of the land, the ratio of carrot to corn is 4 : 1. What is the ratio of carrot to corn for Pat's portion? [11 : 9]

SR-E10 *(25s)* How many possible real numbers y are there so that $x^2 + 2yx + 2013 = 0$ has two integral roots? [8]

Average

SR-A1 *(45s)* Find the sum of all real values of b that satisfy the equations $\begin{cases} a^2b^4 + a^2b^2 + a^2 = 525 \\ ab^2 + ab + a = 35 \end{cases}$. $\left[\frac{5}{2}\right]$

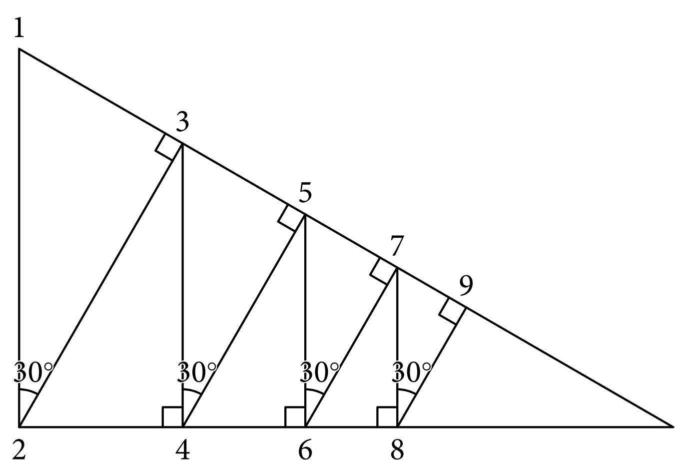
SR-A2 *(30s)* Let P be the sum of all $\binom{8}{p}$, where p is prime, and Q be the sum of all $\binom{8}{q}$, where q is a nonnegative integer which is not prime. Find $P : Q$. [37 : 27]

SR-A3 *(30s)* The function $g(x)$ satisfies the equation $g(x) = g(x-1) + g(x+1)$ for all integral values of x. If $g(1) = 1$ and $g(2) = 3$, what is the value of $g(20) + g(13) + g(2013)$? [6]

SR-A4 *(40s)* Let x be a real number such that $\cot x + \tan x = 3$. Find $\cot^5 x + \tan^5 x$. [123]

SR-A5 *(45s)* Philippine Tournament Bridge Association's new members Jojo, Bogart, Shaniqua, and Chanda May are playing the card game *Bridge*, and they do not know who should be the first dealer. To decide fairly in the Filipino way, they play *maiba*, where each player shows palms up or palms down in a single round, and a dealer is decided when exactly one player shows his or her palm differently—*naiba*—from the others. What is the probability that they will have not have a dealer after three rounds? $\left[\frac{1}{8}\right]$

SR-A6 *(45s)* Sadako is drawing triangles in the process shown in the figure (follow the numbers). If she continues to do this process until the triangles become very small, what is the total area of the larger triangle formed by the infinitely many smaller triangles, given that segment $\overline{56}$ has length $\frac{45}{8}$? $\left[50\sqrt{3}\right]$



SR-A7 *(60s)* The Fibonacci Sequence is the sequence of numbers $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ with each term being the sum of the previous two terms. If the numbers in the sequence are arranged in a triangular array (as shown below), and then replaced with their remainder when divided by 8, what is the 26th term in the 2013th row of the array? [5]

$$\begin{array}{ccccc} & & 0 & & \\ & 1 & 1 & 2 & \\ 3 & 5 & 0 & 5 & 5 \\ & & \vdots & & \end{array}$$

SR-A8 *(30s)* How many 3-digit numbers are there such that the sum of their digits is divisible by 11? [82]

SR-A9 *(60s)* If $\cos x + \sin x = \frac{1}{2}$, find the exact numerical value of $\cos^6 x + \sin^6 x + 2\sin^3 x \cos^3 x$. $\left[\frac{121}{256}\right]$

SR-A10 *(25s)* Vincent and Paula play bato-bato pick, but they have an odd way of scoring. They each start with one point. If Vincent wins a round, the value of Paula's score is added to his score. If Paula wins a round, the value of Vincent's score is added to her score. Otherwise, their score will not be affected. For example, suppose the score is 4-7, in favor of Vincent. If Paula wins the next round, the new score will be 11-7, in favor of Paula. If the score is 50-97, in favor of Vincent, how many times has Vincent won? [17]

Difficult

SR-D1 *(100s)* If A, B, P, and T are four random integers, what is the probability that $PT - BA$ is divisible by 3? $\left[\frac{11}{27}\right]$

SR-D2 *(60s)* Let w, x, y, and z be positive real numbers such that $w < x$, $y < z$, $xy > wz$. Find the area of the triangle with sides $\sqrt{w^2 + y^2}$, $\sqrt{x^2 + z^2}$, and $\sqrt{(x-w)^2 + (z-y)^2}$. $\left[\frac{xy - wz}{2} \text{ units}^2\right]$

SR-D3 *(75s)* A's favorite number is $2^{20}13^{13}$. Determine the number of positive factors of the square of A's favorite number which are less than it but does not divide it. [260]

SR-D4 *(90s)* Determine all the solutions to the equation $\frac{1}{x-1} + \frac{2}{x-2} + \frac{6}{x-6} + \frac{7}{x-7} = x^2 - 4x - 4$.

$$\left[\left\{0, 4, 4 \pm \sqrt{\frac{17 \pm \sqrt{41}}{2}}, 4 \pm \sqrt{\frac{17 \mp \sqrt{41}}{2}}\right\}\right]$$

SR-D5 *(90s)* Two ellipses, $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $E_2 : \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, have four coinciding points. Tangents at each of those points of each ellipse are drawn. The four tangent lines of each ellipse form a quadrilateral that circumscribes the respective ellipses. Find the area of the overlapping parts of the two quadrilaterals. $\left[\frac{2\sqrt{2}a^2b^2}{a^2 + b^2}\right]$

SR-D6 *(150s)* The year 2013 can be referred to as "twenty thirteen". In how many ways can "twenty thirteen" be arranged such that the vowels are fixed, meaning on the same positions, and that no consonant is in the same positions? [5160]

SR-D7 *(75s)* In $\triangle MSA$, $\frac{SA}{MA} = 2 + \sqrt{3}$, $\angle A = 60°$. Find $\angle M$, $\angle S$. $[\angle M = 105°, \angle S = 15°]$

Quarterfinal Round

Easy

QFR-E1 *(30s)* Let $f(x) = 2^{kx} + 9$, where k is a real number. If $f(3) : f(6) = 1 : 3$, find the value of $f(9) - f(6)$. [180]

QFR-E2 *(30s)* What is the remainder when $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + 2013 \cdot 2013!$ is divided by 70? [69]

QFR-E3 *(30s)* Evaluate $\sin 35° \cos 65° - \cos 65° \cos 5° - \cos 55° \cos 5°$. $\left[-\frac{3}{4}\right]$

QFR-E4 *(20s)* In an acute triangle XYZ, $\angle ZXY = 55°$. A is the foot of the perpendicular from Y to XZ, and $\angle AYZ - 18°$. Q is a point on line segment YA such that $\angle QXY = 37°$. Find $\angle QZX$. [35°]

QFR-E5 *(30s)* For how many integers n, with $2 \le n \le 100$, is $\dfrac{n^3 - n}{8}$ an integer? [61]

QFR-E6 *(30s)* 6 dice are rolled. How many possibilities are there such that at most three of the dice show the same number? [2430]

Average

QFR-A1 *(45s)* The numbers a, b, c, in that order, form an arithmetic sequence and $a + b + c = 60$. The numbers $a - 2, b, c + 3$, in that order, form a geometric sequence. Determine all possible of $b - c$. [7, −2]

QFR-A2 *(45s)* What is the sum of all the possible positive integer values of n such that $n^2 + 3n + 21$ is a perfect square? [22]

QFR-A3 *(45s)* Find all pairs (x, y) of real numbers with $0 < x < \frac{\pi}{2}$ such that $\dfrac{(\sin x)^y}{(\cos x)^{y^2/3}} + \dfrac{(\cos x)^y}{(\sin x)^{y^2/8}}$. $\left[\left(\frac{\pi}{4}, 4\right)\right]$

QFR-A4 *(40s)* If the integer part and decimal part of $\sqrt{91 - 48\sqrt{3}}$ are x and y, respectively, find the value of $x + y + xy + 1$. $[21 - 9\sqrt{3}]$

QFR-A5 *(30s)* Justin enjoyed his MSA lessons about factorials and decides to create his ownfunction called the "double" factorial and defines it as $k!! = \begin{cases} k \cdot (k-2) \cdot (k-4) \cdots 3 \cdot 1, & \text{when } k \text{ is odd} \\ k \cdot (k-2) \cdot (k-4) \cdots 4 \cdot 2, & \text{when } k \text{ is even} \end{cases}$. What is the exponent of 7 in the prime factorization of 2013!! ? [169]

Difficult

QFR-D1 *(120s)* An eccentric clockmaker modifies an analog clock such that its hands' speed increases every minute. Since the start of its operation, it moves normally during the 1st minute, but is a second faster during the 2nd minute, and moves another second faster on the 3rd minute, and so on. The clock however moves at normal speed again after each hour in real time, but continues its speed increasing system after each minute. The clockmaker starts the clock at 12:00 MN. What is the correct time by the time the clock has shown 2:45 PM for the 10th time? [7:45 AM]

QFR-D2 *(60s)* The nth MSA number, M_n, is the integer that follows the property $M_n = nM_{n-1} + (n-1)(n-1)!$, for every $n > 0$. What is the remainder of the 2013th MSA number when divided by 2011? [2]

QFR-D3 *(100s)* Obnoxious Ober like to hashtag his activities so much that he is 79% prone to add follow-up hashtags to his original. As he can sometimes get lazy, Obet may include some, all, or none of the possible combinations of words (taken in original order) in hashtags after his original one, as with '#bigredchicken' below with 6 follow-up hashtags.

#bigredchicken #big #red #chicken #bigred #redchicken #bigchicken

The probability that he includes a word in a follow-up hashtag is proportional to the number of letters he has to type. What is the probability that his snack today of #deliciousjumbohotdog (3 words) will have '#delicious #jumbo #jumbohotdog' exactly as follow-up hashtags? $\left[\frac{1393}{3100}\right]$

QFR-D4 *(75s)* A side of the square $ABCD$ lies on the line $y = 4x - 21$ while the vertices of the opposite side are on $y = x^2$. Find the largest possible area of $ABCD$. [68 units^2]

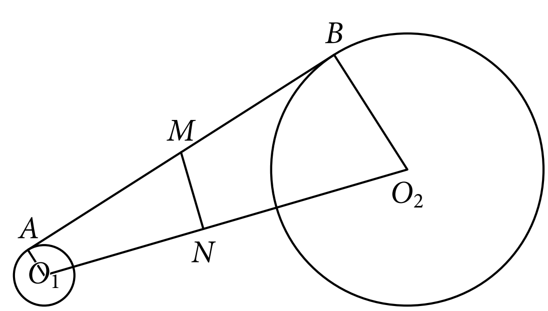
Semifinal Round

Easy

SFR-E1 *(30s)* Vincent rolls three fair six-sided dice. The first die that stops rolling has a 2 on top. What is the probability that after all dice stop rolling, $\gcd(a, b, c) = 1$, where a, b, and c, are the three numbers on the top faces? $\left[\frac{21}{136}\right]$

SFR-E2 *(20s)* Alec draws a sequence of 30°-60°-90° right triangles in the following manner: After drawing the first triangle, he uses the longer leg of this triangle as the hypotenuse of the next triangle. He continues this process and stops when he has drawn 11 adjacent triangles. If the length of the hypotenuse of the first triangle is 1 cm, what is the length of the longer leg of the 11th triangle? $\left[\frac{243\sqrt{3}}{2048}\text{ cm}\right]$

SFR-E3 *(30s)* In the figure, A and B are points of tangency to the circles centered at O_1 and O_2 respectively, $AM = MB = 12$ and N is a point on $\overline{O_1O_2}$ such that the tangents to one circle from N has the same length as the tangents to the other circle. If $AO_1 = 2$ and $BO_2 = 9$, find the length of $\overline{MN}$. $\left[\frac{132}{25}\right]$



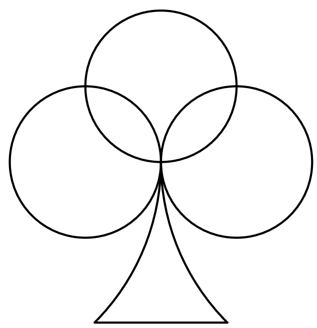
SFR-E4 *(10s)* If m and n are nonnegative integers such that $m < n$, we define $m \bowtie n$ to be the sum of the integers from m to n, inclusively. Determine the value of $\dfrac{(2a-1)\bowtie(2a+1)}{(a-1)\bowtie(a+1)}$ for every positive integer a. [2]

SFR-E5 *(20s)* A positive integer ends with the digit 3. If this digit is moved and becomes the first digit of the number then the number is three times as large as the original number. What are the first three digits of the smallest number satisfying the said conditions? [103]

Average

SFR-A1 *(45s)* Achel's party for the successful lauch of her album "Repahpeepz 08A" had 2013 attendees which were seated around a circular table. The attendees are numbered consecutively in a counterclockwise manner from 1 up to 2013. There are 2013 waiters, also numbered from 1 to 2013. Each waiter has an urn containing three balls, one lettered L, one C, and one R. Each waiter u draws a ball at random from his urn and if the ball is lettered L, delivers a dessert to the attendee to the left of attendee u. If the letter is C, attendee u gets the dessert, and if the letter is R, the attendee to the right of attendee u gets the dessert. Find the probability of having the greatest number of attendees getting three desserts. $\left[\frac{1}{3^{2012}}\right]$

SFR-A2 *(40s)* The club suit can be constructed as follows: the leaves are made up of three circles of equal radius 2 cm, two of which are tangent, and the third one intercepting the first two in arcs of measure 90°. The curved sides of the stem are 45° arcs tangent to each of the first two circles and radius thrice that of the small circles. Find the total area of the club suit. $\left[(36\sqrt{2} - 10 - \pi)\ \mathrm{cm}^2\right]$



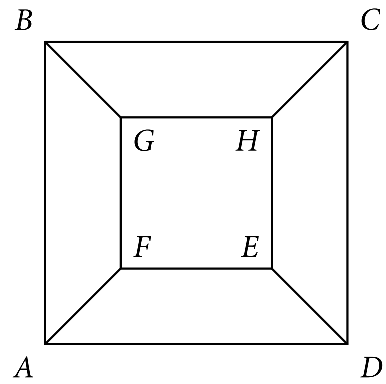
SFR-A3 *(30s)* If m and n are the two solutions of $3x^2 - 6x - 2 = 0$, find the value of $\dfrac{-2m^3 - 2n^3 + 4m^2 + 4n^2}{2m + 2n - mn - 4}$. $[-4]$

SFR-A4 *(30s)* Dabz owns 2014 dove dwellings, arranged in a circles. Each dwelling has a dove numbered from 1 to 2014. One day, one of Dabz's doves says to Dabz that they intend to do the Dabz dove dubstep dance, a dance wherein at each beat of the dubstep, doves step d dwellings to their left, where d is the number assigned to the dove. How many of Dabz's doves will be able to do the Dabz dove dubstep dance in each of Dabz's dove dwellings? [936]

Difficult

SFR-D1 *(120s)* If $\theta = \cot^{-1}(1^2 - 1 + 1) + \cot^{-1}(2^2 - 2 + 1) + \cdots + \cot^{-1}(2013^2 - 2013 + 1)$, what is $\tan^2\theta - \tan\theta + 1$? [4 050 157]

SFR-D2 *(150s)* In the figure, a pair of letters are either connected by a line or not. An *n-dikit* letter sequence is composed of n letters from the figure, with every adjacent letter in the sequence connected by a line; an example of a 5-*dikit* letter sequence is *AFABG*. How many 7-*dikit* letter sequences are there that start with a vowel and end with a vowel? [372]



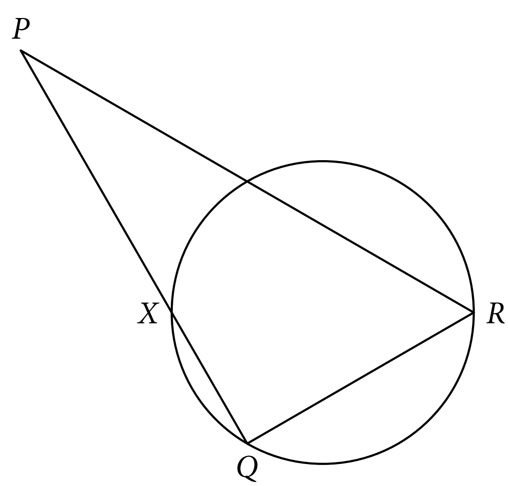
SFR-D3 *(75s)* The sum of the digits of the number 2013 is 6. If all natural numbers, whose digits have sum of 6, listed in increasing order, correspond to $x_1, x_2, x_3, \ldots$, and $x_n = 2013$, find $x_{\lfloor n/2 \rfloor}$. [420]

Final Round

Easy

FR-E1 *(30s)* In a basketball league with 6 teams (A, B, C, D, E, F), each team must eventually play all other teams exactly once. So far, Team A has played one match, Team B has played 2 matches, Team C has played 3 matches, Team D has played 4 matches, and Team E has played 5 matches. How many matches has Team F played so far? [3]

FR-E2 *(30s)* In the figure below, O is the center of the circle, $\angle ORQ = \angle ORP = \frac{1}{3}\angle PQR = 30°$. If $PX = \sqrt{23}$, find the length of the segment of PR not contained in the circle. $\left[\frac{\sqrt{69}}{2}\right]$



FR-E3 *(20s)* Given that $x + \sqrt{x^2 - 1} + \dfrac{1}{x - \sqrt{x^2 - 1}} = 20$, find the exact value of $x^2 + \sqrt{x^4 - 1} + \dfrac{1}{x^2 + \sqrt{x^4 - 1}}$. $\left[\frac{10\,201}{200}\right]$

FR-E4 *(30s)* Given that x and y are positive integers such that $\dfrac{1}{x} + \dfrac{1}{2x} + \dfrac{1}{3x} = \dfrac{1}{y^2 - 2y}$, find the smallest possible value of $x - y^2$. [8]

FR-E5 *(35s)* Find the sum of all the prime factors of $N = \dfrac{2014^2 - 1234^2 - 780^2}{8}$. [640]

Average

FR-A1 *(60s)* Find all $x \in [0, 2\pi)$ which satisfies $\tan x + \tan 2x + \tan 3x = 2013 \tan x \tan 2x \tan 3x$.
$\left[0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, \cot^{-1}\sqrt{2015}\right]$

FR-A2 *(30s)* The equation $4^{671x+1} + 1 = 2^{671x-4} + 2^{2013x-(1/2)}$ has three roots whose sum is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$. [1343]

FR-A3 *(45s)* Evaluate $1 + \frac{2}{4} + \frac{5}{16} + \frac{13}{64} + \cdots + \frac{F_{2k}}{4^k} + \cdots$, where F_n denotes the nth term in the sequence defined as $F_0 = F_1 = 1$; $F_n = F_{n-1} + F_{n-2}$. $\left[\frac{12}{5}\right]$

FR-A4 *(45s)* Ten employees of People 360° need to group themselves to be able to ride identical minivans. Each minivan can seat up to three people. Nobody wants to be left alone in a minivan, so they need to form groups of two or three. How many ways can they do this? [7245]

FR-A5 *(45s)* Determine the product of the maximum and minimum value of $\frac{x - y}{1 + xy}$, where $\frac{1}{6} \le x, y \le 6$. $\left[-\frac{1225}{144}\right]$

Difficult

FR-D1 *(120s)* Find the least natural number that satisfies the following inequality $\sqrt{\frac{n-2012}{2013}} - \sqrt{\frac{n-2013}{2012}} < \sqrt[4]{\frac{n-2014}{2012}} - \sqrt[4]{\frac{n-2012}{2014}}$. [4026]

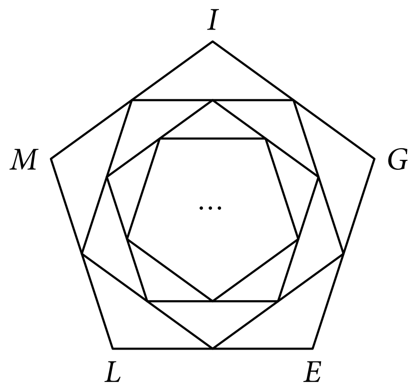
FR-D2 *(60s)* A square of side length s is inscribed symmetrically inside a sector of a circle with radius of length r and central angle of 60°, such that two vertoces lie on the straight sides of the sector and two vertices lie on the circular arc of the sector. Determine the exact value of $\sqrt{\frac{s}{r}}$. $\left[\sqrt[4]{2 - \sqrt{3}}\right]$

FR-D3 *(90s)* The value of the expression

$$\frac{1}{\sqrt[5]{1 + \left(\sqrt[5]{1 + (\sqrt[5]{1 + \cdots})^4}\right)^4}}$$

is the real root of $P(x)$, a polynomial with leading coefficient 1. Determine $P(x)$.
$[P(x) = x^3 + x^2 - 1]$

FR-D4 *(100s)* Miguel's favorite shape is the regular pentagon, and he is interested in manipulating the figure below, where the vertices of each pentagon beginning with the second lie on the midpoints of the next bigger one, continuing indefinitely. If he wishes for the sum of the areas of all pentagon to be equal to 26 cm², what should be the sidelength of the largest one? $\left[\sqrt{\frac{26 \tan 36°}{5}}\ \text{cm}\right]$



FR-D5 *(90s)* In an attempt to poison her stepdaughter Valentine, Heloise pours a certain chemical into a drink prepared for the former's refreshment. The poison, however, is slow-acting, and will require some 40 minutes to attain full lethality. Of course, Heloise is a cautious woman, and has added twice the amount necessary to kill the innocent lady. Maximilian, seeing the deed, attempts to save his lover by adding another chemical to te brew, one which slows the speed of the first by a third. Little Edward, for whom the murder is attempted, stumbles upon the container 5 minutes later and proceeds to agitate it, increasing the speed at which the poison gained lethality by 20% of its current value. Maximilian returns another 5 minutes later with a brew that halts the poison and counteracts the lethality, but which will take 20 minutes to remove the intended lethality. If Valentine drinks the brew exactly 45 minutes after Heloise's actions but does not die, determine the greatest possible elapsed time between Heloise's and Maximilian's actions. $\left[\frac{248}{9}\text{ minutes}\right]$