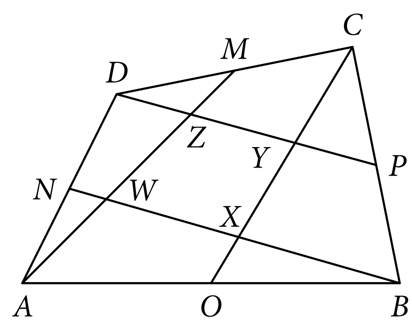
Survival Round

Easy

SR-E1 *(15s)* Princess Sarah and Becky are peeling potatoes. Given the same number P of potatoes, Princess Sarah can finish peeling them alone 2 hours and 36 minutes before Becky can, alone. If Princess Sarah is given $P + \frac{P}{2}$ potatoes while Becky is given $\frac{9P}{10}$ potatoes, they finish at the same time if they both work alone. Both girls work at constant rates. If Becky helps Princess Sarah peel 2015 potatoes, how many minutes does Princess Sarah save? [26 minutes]

SR-E2 *(15s)* Let a 5 digit number be termed a "valley" number if the digits (not necessarily distinct) in the number $\overline{abcde}$ satisfy $a > b > c$ and $c < d < e$. Compute the number of valley numbers that start with 4. [185]

SR-E3 *(20s)* As shown in the figure, $ABCD$ is a convex polynomial with M, N, O, and P as the midpoints of its sides. $WXYZ$ is the quadrilateral formed by the intersections of AM, BN, CO, and DP. Given that the areas of $ABCD$, $AWXO$, and $CMZY$ are 1500 units^2, 308 units^2, and 227 units^2, respectively, find the area of $WXYZ$.* [215 units^2]



SR-E4 *(15s)* We define a positive integer p to be *almost prime* if it has exactly one divisor except 1 and p. Compute the sum of the first 5 numbers which are almost prime. [208]

SR-E5 *(15s)* Norman and Vincent's grades always differ by at most 5.00. Otherwise, they become suspicious of their teacher's grading style. One day, Ms. Forgetful lost all the final exam papers of her students. She was so agitated, she decided to give her students real number grades randomly chosen from 0 to 100. What is the probability that Ms. Forgetful does not attract suspicion from Norman and Vincent? $\left[\frac{39}{400}\right]$

SR-E6 *(30s)* If $\log_{49} x = \log_{28}(y\sqrt{2015}) = \log_{16}(x + 2y)$, find the numerical value of $\frac{x}{y}$. $[12\sqrt{14} - 1]$

SR-E7 *(20s)* If $\sin 2x = \frac{2}{3}$, what is the value of $\sin^6 x + \cos^6 x$? $\left[\frac{2}{3}\right]$

SR-E8 *(10s)* Nika likes to think of numbers. One time, she thought of the number five. While daydreaming, she decided to do four things consecutively. First, she added one to the number five. Then, she multiplied the number four to the sum. Next, she subtracted six from the product. Finally, she divided the difference by two. If this set of steps were done repeatedly, the first step of the next set

* Figure not drawn to scale. The answer implies that $[WXYZ]$ is less than both $[AWXO]$ and $[CMZY]$. I do not believe this is possible.

would be to add one to the quotient of the last step of the set before. What is the answer after she did 2015 sets? $[2^{2017}+1]$

SR-E9 *(25s)* Let $f(x) = x^5 + 3x^2 + 27$ have roots r_1, r_2, r_3, r_4, r_5. Let $g(x) = x^2 - 3$. What is the value of $g(r_1)g(r_2)g(r_3)g(r_4)g(r_5)$? $[1053]$

SR-E10 *(30s)* Find the last two digits of $\sum_{n=1}^{2015} n! + \sum_{n=1}^{2014} n! + \cdots$. $[09]$

Average

SR-A1 *(30s)* Brian and Daniel are playing a game. Brian tells Daniel, "I am thinking of a polynomial function $f(x)$ with nonnegative integer coefficients." On a whim, Daniel asks, "What is $f(11)$?", to which Brian replies, "29 669". Daniel asks once more, "What is the sum of the coefficients?" to which Brian replies "9". After a few minutes, Daniel realizes what the polynomial $f(x)$ is. Determine $f(2)$. $[50]$

SR-A2 *(45s)* Russ is tired of Math questions that ask for the probability of the sum of dice values. Instead he does this: he rolls three fair dice and notes that the probability that sum of the faces of the dice is equal to X is $\frac{7}{72}$. What is the product of all possible values of X? $[104]$

SR-A3 *(40s)* Given a number N with exactly 9 factors, what is the sum of the factors whenever $N \leq 10\,000$ and $N - 1$ is divisible by 6? $[1333]$

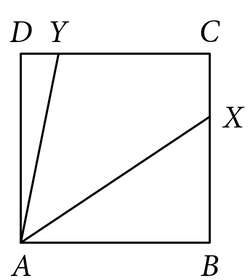
SR-A4 *(40s)* Patrick the Jejelord is tired of the usual "odd" or "even" numbers. Instead, he decides to call a number Eyoh, Poze, or Acheje if it leaves a remainder of 1, 2 or 0 respectively when divided by 3. Let A be the set of all multiples of 7 in $[1, 2015]$, while B is the set of integers in $[1, 2015]$ that are not multiples of 8. Determine the number of all ordered pairs (a, b) such that $a \in A$, $b \in B$, and $a + b$ is Poze. $[168\,756]$

SR-A5 *(45s)* $\triangle ABC$ has side lengths $AB = 13$, $AC = 15$, and $BC = \sqrt{58}$. Find $\tan \angle BAC$. $\left[\frac{33}{56}\right]$

SR-A6 *(40s)* Let $f(x)$ be a polynomial function with degree 3 and positive integral coefficients less than 10. If $f(1) = 16$, how many possible values are there for $f(10)$? $[66]$

SR-A7 *(45s)* Find the sum of all real values of x such that $\frac{8x^2+16x+2015}{3x^2+x+2010} = \frac{9x^2+15x+2020}{4x^2+2015}$. $[-3]$

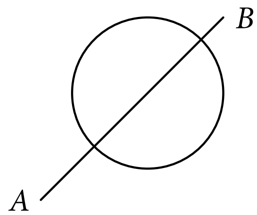
SR-A8 *(40s)* In the figure below, $ABCD$ is a square, $AD = 5$, $DY = 1$, and $\tan \frac{\angle YAX}{2} = \sqrt{2} - 1$. Find BX. $\left[\frac{10}{3}\right]$



SR-A9 *(45s)* Let $1 + 11 + 111 + \cdots + \underbrace{111\cdots11}_{2015}$, where the last term has 2015 digits, each equal to 1. Find

$\left\lfloor \frac{10^{2015}}{S} \right\rfloor$. [8]

SR-A10 *(45s)* Joma and his babygirl JL are strolling in a park when they spot a circular plot with a sign "no stepping on the grass" 2 m away from them. If the diameter of the plot is 4 m and they want ot go to the MSA study center at point B, which is $(4+2\sqrt{2})$ m from where they stand at point A, find the shortest distance they need to travel without breaking the law. $\left[\left(2\sqrt{3}+\frac{5\pi}{6}+2\right) \text{ m}\right]$



Difficult

SR-D1 *(60s)* Junior is listing a sequence $2, 3, 5, 6, 7, 8, 10, \ldots$ that consists of positive integers which are not perfect squares. Thirdie is also listing a sequence $2, 3, 4, 5, 6, 7, 9, 10, \ldots$ that consists of positive integers which are not perfect cubes. What is the positive difference between the 2015th terms of the two sequences? [32]

SR-D2 *(90s)* An integer is said to be a palindrome if it can be read the same way by reversing the order of its digits. Examples of palindromes would be 12 321, 444, 69 696, and many more. Let N be an odd natural number such that the sum of all N-digit palindromes is $49\,500\,000\,000 \cdots 000$, which has 2015 zeroes. Determine the value of N. (For this problem, we will not consider 01 210 as a palindrome, as do all other numbers with zero as its first and last digit.) [1345]

SR-D3 *(70s)* In square $ILAX$ with center O, a point P is randomly chosen in its interior. What is the probability that $\triangle PAL$ is an acute triangle and that the length $2PO \leq AL$? $\left[\frac{2\pi+3\sqrt{3}}{24}\right]$

SR-D4 *(50s)* Let w, x, y, z be real numbers satisfying the conditions $w+x+y+z=9$, $xy+yz+xz+wx+wy+wz=27$. Find the largest possible value of w. $\left[\frac{9}{2}\right]$

SR-D5 *(60s)* Rachelle and her new friend, Jessa the Chocolate Thief, eat their shared stash of 100 chocolates in a weird way. On some day of the week that they randomly picked, they start eating. Each of them eats one chocolate per day every day until they decide to stop eating their share. Once one of them stops eating, the other continues eating the rest in the same one-chocolate-per-day diet until they finish. What is the probability that Rachelle eats her last chocolate on a Tuesday while Jess eats her last chocolate on a Thursday? $\left[\frac{2}{99}\right]$

SR-D6 *(75s)* One day, Stephen was bored and decided to find amazing polynomial functions. He found one such polynomial function $S(x)$, which he calls Stephen's function, such that the degree of $S(x)$ is n, and for all $k \in \{0, 1, 2, \ldots, n-1, n\}$, $S(k) = \frac{k}{k+2}$. If $S(n+2) = \frac{45}{47}$, find the value of Stephen's function at $x = n$. $\left[\frac{45}{46}\right]$

SR-D7 *(60s)* Evaluate $\sum_{n=1}^{2015} \frac{5^{2016}}{25^n + 5^{2016}}$. $\left[\frac{2015}{2}\right]$

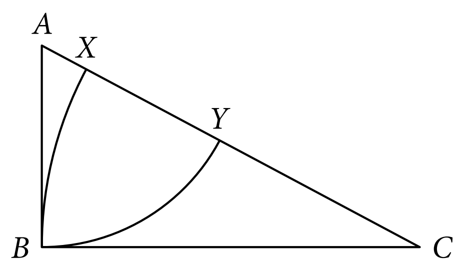
Quarterfinal Round

Easy

QFR-E1 *(25s)* The sides of a triangular prism with dimensions $5 \times 13 \times 31$ are painted in such a way that both its 5×13 sides are colored red, both 5×31 sides are colored blue, and both 13×31 are colored green. Then, the prism is divided into 2015 smaller congruent cubes each with side length 1. All cubes that have at least one side colored are placed inside a jar. If one cube is to be randomly selected from the jar, what is the probability that the cube selected will have a green-colored or a blue-colored side? $\left[\frac{182}{193}\right]$

QFR-E2 *(15s)* Find the sum of all value(s) of b such that $\dfrac{11}{\log_2 x} + \dfrac{1}{2\log_{25} x} - \dfrac{3}{\log_8 x} = \dfrac{1}{\log_b x}$ for all $x > 1$. $[20]$

QFR-E3 *(20s)* $\triangle ABC$ is a right triangle and the curves BX and BY are arcs of circles centered at C and A, respectively. If $AX = 2\,\text{cm}$ and $CY = 9\,\text{cm}$, what is the area of $\triangle ABC$? $[60\,\text{cm}^2]$



QFR-E4 *(25s)* Lara-Lara hated things that are too many including the fact there are so many digits. So, she decided to use only digits 0 to 3 such that her Laran sequence is increasing order is 0, 1, 2, 3, 10, 11, 12, 13, 20, 21, ... She calls these numbers Laran numbers, which are denoted by a subscript L. How many positive integers a less than or equal to 2015 satisfy the equation $(a_L)^2 = (a^2)_L$? $[14]$

QFR-E5 *(20s)* Solve for the value(s) of x that satisfy $\tan(120° - x) = \dfrac{\sin 120° - \sin x}{\cos 120° - \cos x}$, where $0° \le x < 360°$. $[100°, 220°, 340°]$

QFR-E6 *(15s)* Find the last digit of $2^{3^{4^{\cdots^{2015}}}} - 2015^{2014^{2013^{\cdots^{2}}}}$. $[7]$

Average

QFR-A1 *(45s)* Let $a, b \in \mathbb{N}$ such that a and b are relatively prime. If the sum of all positive divisors of a is 2014 and the sum of all positive divisors of b is 2016, find the sum of all positive divisors of ab. $[4\,060\,224]$

QFR-A2 *(30s)* Let $ABCD$ be a square of area $(8\sqrt{3} + 12)\,\text{cm}^2$ with point E in its interior such that $\triangle ADE$ is equilateral. Let F be the point of intersection of AE and BD. Find the area of $\triangle FED$. $[3\,\text{cm}^2]$

QFR-A3 *(30s)* Find $2015x$ if $x = \dfrac{1}{2015+}\,\dfrac{1}{x+}\,\dfrac{x}{1+}\,\dfrac{1}{x+}\,\dfrac{x}{1+}\,\dfrac{1}{x+}\,\dfrac{x}{1+}\cdots$. $\left[\frac{2014}{2013}\right]$

QFR-A4 *(40s)* Let $f(x)$ and $g(x)$ be 2015th degree polynomial functions that satisfy $f(n) = g(n)$ for $n = 1, 2, 3, \ldots, 2015$. If $f(2016) = g(2016) + 2015$, find the value of $\dfrac{f(2017) - g(2017)}{2015}$. $[2016]$

QFR-A5 *(60s)* In a seminar room that has a rectangular array of chairs, writers from the Manila Bulletin and staff from MSA are attending a talk. There must be exactly 14 writers from Manila Bulletin in each row and exactly 10 staff members from MSA in each column. If there are 3 seats that should be empty, what must be the minimum number of chairs inside the seminar room? [567]

Difficult

QFR-D1 *(60s)* From his custom deck containing one 1, two 2s, three 3s, ..., ten 10s, eleven jacks, twelve queens, and thirteen kings, James Brian randomly picks up two cards without replacement. If he sees four male faces, what is the probability that he picks up a *king* on his next draw? $\left[\dfrac{143}{1068}\right]$

QFR-D2 *(90s)* Find the smallest natural number n such that $n!$ ends with 2014 zeroes. [8070]

QFR-D3 *(75s)* In square $HERB$, C is a point in its interior such that $\overline{BC}$ and $\overline{EC}$ are angle bisectors of $\angle HBE$ and $\angle HEB$ respectively. If circle A of radius $(2+\sqrt{2})^{3/2}$ cm is tangent to $\overline{BC}$ at D and to $\overline{EC}$ at F, where D and F are midpoints of $\overline{BC}$ and $\overline{EC}$ respectively, find the area of the square $HERB$. $[2\,\text{cm}^2]$

QFR-D4 *(60s)* Let $f(x) = \dfrac{8x+\sqrt{16x-4}}{\sqrt{4x+2}+\sqrt{4x-2}}$. Determine the value of $\sum\limits_{x=1}^{144} f(x)$. $\left[2456\sqrt{2}\right]$

Semifinal Round

Easy

SFR-E1 *(30s)* In trapezoid $ABCD$, with $AD \parallel BC$, E is the intersection of diagonals AC and BD. If the area of $\triangle ABD$ is $25\,\text{units}^2$, the area of $\triangle ABC$ is $29\,\text{units}^2$, $BC=\sqrt{12}$, and $AD=\sqrt{10}$, find the area of $\triangle DEC$. $[5\,\text{cm}^2]$

SFR-E2 *(20s)* If a fair six-sided die is rolled five times, what is the probability that exactly three rolls show a composite number? $\left[\dfrac{40}{243}\right]$

SFR-E3 *(20s)* A digit slide is performed on a number by moving its unit digit to the front of the number. For example, the result of a digit slide on 1027 is 7102. What is the smallest positive integer N such that the units digit of N is 8 and the result of a digit slide on N equals four times N? [205 128]

SFR-E4 *(25s)* Among the times when the minute hand of a 12-hour clock overlaps with the other hand, at which two times, other than 12:00, are the second hand the closest? Round off your answer to the nearest minute. [3:16 and 8:44]

SFR-E5 *(30s)* What is the maximum value of c such that the ellipse $4x^2+y^2=16$ touches the parabola $y=cx^2-4$ at a unique point? $\left[\dfrac{1}{2}\right]$

Average

SFR-A1 *(30s)* The Ackermann-Péter function is defined for nonnegative integers m, n as

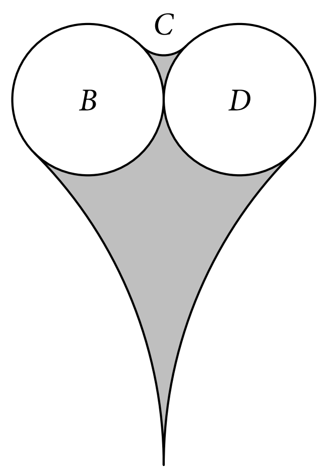
$$A(m,n)=\begin{cases} n+1, & \text{if } m=0 \\ A(m-1,1), & \text{if } m>0 \text{ and } n=0 \\ A(m-1,A(m,n-1)), & \text{if } m>0 \text{ and } n>0 \end{cases}.$$

What is the value of $A(1,2)\cdot A(2,1)$? [20]

SFR-A2 *(60s)* The Manila Bulletin has the following problem posted on their Puzzles page: "A special sequence is defined weirdly, being dependent on the term **after** it. The nth term is $a_n = \dfrac{a_{n+1} - 15}{2}$. If $a_1 = 1$ and $a_{2015} = 2^k - 15$, find the value of k." [2018]

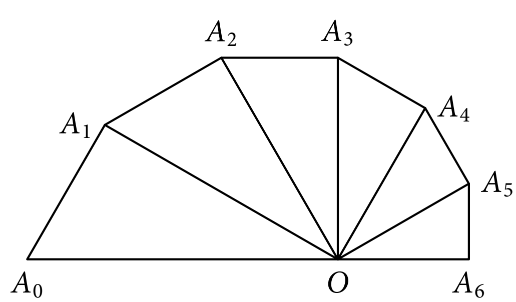
SFR-A3 *(60s)* Evaluate $A = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \cdots + \sqrt{1 + \frac{1}{2014^2} + \frac{1}{2015^2}}$.
$\left[2015\frac{2015}{2016} \text{ or } \frac{4\,064\,255}{2016}\right]$

SFR-A4 *(45s)* Hirai, using her engineering skills, draws a *haato*, where all curved lines are circular arcs which are tangent at their points of contact. The centers of each arc are indicated below. The radius of each white circle is 1 cm and the arc with center C measures 90°. If the area of the blackened part of Hirai's *haato* is $\left((a + b\sqrt{2}) - \frac{\pi}{4}(c + d\sqrt{2})\right)$ cm² for integers a, b, c, d, where c and d are not divisible by 4, find $a + b + c + d$. [53]



Difficult

SFR-D1 *(100s)* The figure below contains 6 similar right triangles, with a right angle of some triangle lying on each of $A_1, \ldots, A_6$. If A_0, O, and A_6 are collinear, find the ratio $\dfrac{A_0A_1 + A_1A_2 + A_2A_3 + \cdots + A_5A_6}{OA_0 + OA_1 + OA_2 + \cdots + OA_6}$. [5735]



SFR-D2 *(60s)* "The greatest trick the devil ever pulled was convincing the world he didn't exist," is a quote from the movie *The Usual Suspects*. What is the largest prime factor of the number of ways you can arrange all fourteen words of the quote including the three "the"s such that the words having the same number letters are not grouped altogether as one? The following sequence indicates the letter count of each word in the quote: three, eight, five, three, five, four, six, three, ten, three, five, two, five, five. [257]

SFR-D3 *(75s)* Given a real number $x \in [0, 2\pi)$ satisfying $\cos(x) + \cos(3x) = \cos(x)\cos(2x)$, find the sum of all possible values of $1 - \frac{1}{2}\cos^2\frac{x}{2} + \frac{1}{4}\cos^4\frac{x}{2} - \frac{1}{8}\cos^6\frac{x}{2} + \frac{1}{16}\cos^8\frac{x}{2} + \cdots$. $\left[\frac{128}{245}\right]$

Final Round

Easy

FR-E1 *(25s)* DJs Judy and Homer of Magic 89.9 were talking about "squarey-square" numbers on their radio show, "Mathiradio". These numbers are numbers with at least two digits, such that if you take any two consecutive digits of this number, those two digits are a perfect square. Find the sum of the smallest and largest "squarey-square" numbers. [81665]

FR-E2 *(20s)* Miguel has a biased coin such that if you flip it 7 times, the probability of having 2 heads and 5 tails is $\frac{5}{12}$ of the probability of having 4 heads and 3 tails. If p is the probability that the coin will land on heads, and can be expressed as $\frac{m}{n}$, where m and n are relatively prime, find $2m + n$. [23]

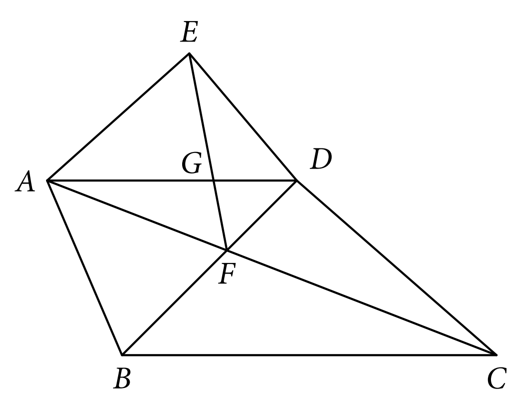
FR-E3 *(20s)* The roots of the equation $x^2 - 9x - 39 = 0$ are m and n. Evaluate $\cos(\arctan m + \arctan n)$. $\left[\frac{40}{41}\right]$

FR-E4 *(25s)* Let $P(x) = x^3 + x^2 + ax + 1$. Find all rational values of a such that $P(x)$ is a rational number for all x satisfying the equation $x^2 - 6x + 2 = 0$. $[-40]$

FR-E5 *(20s)* A right triangle on the Cartesian plane is positioned such that its legs are on the positive axes. Two one-dimensional paint rollers with infinite length are placed parallel to each other. The first roller is placed on the y-axis, while the second is placed in such a way that it intersects exactly one vertex of the right triangle. If the first roller paints the triangle at a rate of one-sixth of the area of the triangle per hour by moving to the right while the second roller paints the triangle by moving to the left at a rate of one-third of the base length per hour, what is the ratio of the area painted by the first roller to the area painted by the second roller when they meet? [1 : 1]

Average

FR-A1 *(60s)* In trapezoid $ABCD$ (with $AD \parallel BC$), diagonals AC and BD are drawn, meeting at point F. Triangle ADE is drawn such that $\angle AEF = 2\angle DEF = 2x$. Additionally, $DE = AE\cos x$ and EF intersects AD at G. If the area of $\triangle ABD$ is $35\,\text{cm}^2$, and $\frac{AD}{BC} = \frac{2}{3}$, find the area of $\triangle FGD$. $\left[\frac{14}{3}\,\text{cm}^2\right]$



FR-A2 *(60s)* Jen received her results on the final exam and found that she was given 7 points out of 10 on the 5-item Right Minus Wrong Modified True or False MSA quiz, where an incorrect statement requires a modification to make it correct. Points and deductions are given accordingly depending on what Jen answers, according to the table below. How many possible combinations of Jen's answers are there? [440]

Jen's Answer	Correct Answer	
	TRUE	FALSE, Modification = X
TRUE	2	−2
FALSE, Modification $Y = X$	−2	2
FALSE, No Modification	−1	1
FALSE, Modification $Y \neq X$	−2	0
BLANK	0	0

FR-A3 *(60s)* Find all values of M such that $\tan M \cot \frac{7\pi}{10} = \cot \frac{\pi}{5} \cot \frac{7\pi}{10} - \cot \frac{\pi}{5} \cot \frac{9\pi}{10} - 1$, M restricted to $[0, 2\pi)$. $\left[\left\{\frac{2\pi}{5}, \frac{7\pi}{5}\right\}\right]$

FR-A4 *(45s)* Sir Rahman Marpheel posed the following question in his Math column in the Manila Bulletin. One can "rewrite" the function $f(x) = 2x^2 - x - 3$ into a different form such as $f(x) = 2(x-1)^2 + 3(x-1)^1 - 2(x-1)^0$. Now consider the infinite sum $\sum_{j=0}^{\infty}\left(\frac{x}{3}\right)^j$. If one rewrites it in the form $a_0 + a_1\left(x+\frac{1}{2}\right)^1 + a_2\left(x+\frac{1}{2}\right)^2 + \cdots = \sum_{j=0}^{\infty} a_j\left(x+\frac{1}{2}\right)^j$, what is the value of $a_1 + a_3 + a_5 + \cdots = \sum_{j=0}^{\infty} a_{2j+1}$? $\left[\frac{4}{15}\right]$

FR-A5 *(45s)* There is a 50% chance that a 65 year old person will live for at least another 10 years, and there is also a 20% chance that he will live for at least another 15 years. A 70 year old person has a 25% chance of living at least another 10 years. Find the probability that a 70 year old person will live for at least another 5 years. [62.5%]

Difficult

FR-D1 *(120s)* Evaluate $\frac{1}{2\cdot3\cdot4\cdot5\cdot6} + \frac{1}{3\cdot4\cdot5\cdot6\cdot7} + \frac{1}{4\cdot5\cdot6\cdot7\cdot8} + \cdots$. $\left[\frac{1}{480}\right]$

FR-D2 *(75s)* Let $\alpha = \cos 1^\circ$ and $\beta = \sin 1^\circ$. Express the value of $\sum_{x=0}^{90}(\cos x^\circ + \sin x^\circ)$ in terms of α and β. $\left[1 + \frac{\beta}{1-\alpha}\right]$

FR-D3 *(75s)* Let a, b, c, and d be real numbers such that $a + 9b + 7c + 4d = \sqrt{\frac{2014}{2015}}$. The minimum value of $5a^2 + 27b^2 + 7c^2 + 40d^2$ can be expressed as $\frac{m}{n}$, where m and n are integers that are relatively prime to each other. What is the value of $n - m$? [365]

FR-D4 *(120s)* A 5 or 6 digit number is an MSA number if its digits are a rearrangement of the numbers 0, 1, 2, 3, 4, 5, 6 with each digit appearing only once. The smallest MSA number is 0 123 456. If the MSA numbers were written in a sequence in increasing order, what is the 1234th MSA number? [1523460]

FR-D5 (90s) E, G, H, I, N, O, P, T, and Y are distinct one-digit positive integers. If the products $GO = 27$, $YIN = 60$, $HE = 28$, $ONE = 216$, $HIP = 280$, $TY = 2$, and $CAM = 190$, what is $MAGIC + EIGHTY - NINE - POINT - NINE$? [90]