1 A number n is *triangular* when you can stack n dots to form an equilateral triangle. How many triangular numbers less than 1 000 000 are there? (Consider 0 to be triangular.) [1414]

2 Let the set $S = \left\{\log_5 4, \dfrac{\sqrt{2016}}{2016}, \log_{125} 27, \log_3 2, \dfrac{2}{3}\right\}$. Find an ordered pair (x, y) where x and y are the smallest and greatest elements of S, respectively. $\left[\left(\dfrac{\sqrt{2016}}{2016}, \log_5 4\right)\right]$

3 Six identical square pyramids with edges of equal length are to be glued together to form one bigger polyhedron by gluing faces of the same area. How many times bigger is the surface area of the arrangement with the most surface area compared to that with the least surface area? $\left[\dfrac{7}{4} + \sqrt{3}\right]$

4 How many three-digit numbers with distinct digits are there such that the sum of its digits is prime? [212]

5 Consider a circle λ with center O and radius r, four equally-spaced points A, B, C, and D on its circumference, four congruent isosceles right triangles $\triangle AEF$, $\triangle BGH$, $\triangle CIJ$, and $\triangle DLK$ such that $\overline{EF}$, $\overline{GH}$, $\overline{IJ}$, and $\overline{LK}$ are parallel to the tangents at A, B, C, and D respectively. Define midpoints M of $\overline{EF}$, N of $\overline{GH}$, Q of $\overline{KL}$, and P of $\overline{IJ}$. What is the area of the octagon defined by the points M, F, G, P, J, K, and Q if the length of $\overline{CD}$ is $\sqrt{2}$ units and $\overline{OC}$ is half as long as $\overline{AM}$? [130 units^2]

6 Determine the length of the longest side of the region on the Cartesian plane bounded by the graphs of the equations $3y - 13 - x = 0$, $17 + y - 11x = 0$, $9x + 21 + 5y = 0$. $[\sqrt{122}]$

7 Goku is trying to buy the following materials for his Mythril Sword. Diamond costs 30 rubies, 30 sapphires, and 30 emeralds; Pearl costs 20 rubies, 30 sapphires, and 40 emeralds; and Platinum costs 50 rubies, 30 sapphires, and 10 emeralds. Goku decides to mine for these gems in the cave for a few days before going back to town to buy all the materials. However, mining is hard work for one day, so Goku only has the following options:

1) he can mine in the entire cave to find 1 of each gem, or

2) he can mine in one specific area of the cave to find 3 of only one gem.

If doing one of the actions above spends one day, what is the minimum number of days that Goku mines before finding all the required gems? [91 days]

8 a, b, c, d, e, f, g, h, i, and j are distinct integers taken from the set $\{8, 14, 20, 36, 44, 59, 67, 78, 88, 93\}$. If $ac + eg + hj = 11\,423$ and $(h+b)(i+f)(j+f) = 648\,225$, what is $a+b+c+d+e+f+g-h-i-j$? [69]

9 The Blood Bank exchanges blood per 50 cc bag at rates shown below.

	BUYS (per bag)	SELLS (per bag)
Type AB+	5 USD	8 USD
Type O−	3 USD	4 USD
Other Types	Not Available	Not Available

Call an integer amount in USD as 'exchangeable' if all of it can be spent to buy bags of blood. (For example, 12 USD is exchangeable because it can be spent to buy a bag of Type AB+ blood and a bag of Type O− blood.) Call an exchangeable amount as 'twice exchangeable' if when spent to buy bags of blood, it can be sold back to an exchangeable amount. (For example, 16 USD is twice exchangeable because it can be spent to buy 0 bags of Type AB+ blood and 4 bags of Type O− blood, and when sold is 12 USD which is

exchangeable.) If the bank allows only 10 bags (not necessarily same type) per transaction, what is the largest twice exchangeable amount of USD? [76]

10 In the fighting game Ketten, pressing a button (out of 5 different buttons) is considered a hit. Pressing two buttons at the same time is also considered a hit. To make a 10-hit combo, the player must make 10 consecutive hits in the following manner:

- a button cannot be pressed consecutively (i.e. after pressing the X button, the next hit cannot contain the X button), and
- a 2-button hit occurs exactly 4 times — it occurs every 3rd consecutive hit and on the last hit.

After playing for 10 straight hours, Bob broke a button. What fraction of the possible 10-hit combos can he no longer execute? $\left[\frac{159}{160}\right]$

11 Rus has a deck of N cards that is labeled in such a way that the numbers 1 through N are put on the cards in order, with card 1 on top, followed by card 2, and so on, with card N at the bottom. Rus has discovered a new way of shuffling cards. We define a YMG Shuffle as follows. Take the top card of the deck, place the next card under it, then the 3rd card on top of the new pile, then the 4th card under the pile, and so on, until all the cards in the deck have been transferred to the new pile, with card N now newly placed at the bottom if N is even, or card N is on top of the pile if N is odd. For example, a YMG Shuffle on a deck of 5 cards $(1, 2, 3, 4, 5)$ will change the order to $(5, 3, 1, 2, 4)$. After playing with the N-card deck using te shuffle for some time, Rus has noticed that a specific card in the deck will always stay fixed, meaning no matter how many times a deck is subjected to the YMG Shuffle, the card stays in the same position always. In particular, it's the card labelled "2015". Find the sum of all possible values of N. [12 087]

12 During a commercial break, Magic 89.9 DJ Tanya Marcucio doodles some triangles in a particular way.

a. He draws all lines with 1 unit length.

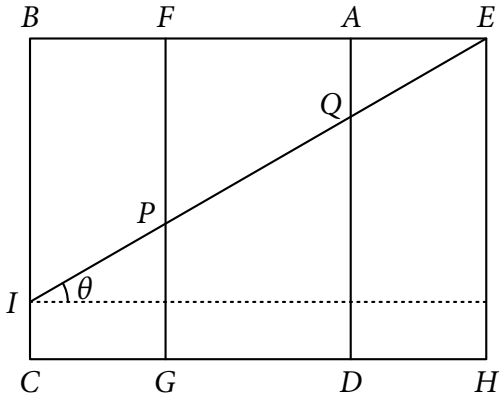
b. He starts with a line ℓ_1 with an initial point P_0.

c. He draws a line ℓ_1 and ℓ_2 forming a $0.5°$ angle at point P_1. He draws ℓ_3 on the other end of ℓ_2 forming a $1°$ angle at point P_2. He draws ℓ_4 on the other end of ℓ_3 forming a $1.5°$ angle at point P_3. In general, he draws ℓ_{i+1} on the other end of ℓ_i forming a $0.5 \cdot i°$ angle at point P_i, for $i = 1, \ldots, 180$.

After finishing his zigzag lines, he connected P_1 to P_3 to form a triangle with area $\triangle_1$, P_3 to P_5 to form a triangle with area $\triangle_2$, P_5 to P_7 to form a triangle with area $\triangle_3$, and he continues the process until he makes 90 triangles. Let $S = \{1, 2, 3, \ldots, 89, 90\} \setminus \{4, 8, 12, \ldots, 80, 84, 88\}$. Find $\mathbb{D} = \log_2 \prod_{i \in S} \triangle_i$.

$\left[-\frac{269}{2}\right]$

13 $ABCD$ and $EFGH$, both wide side length $3\sqrt{2}$ m, is positioned in the manner shown below. A line, $30°$ with respect to the horizontal, is drawn from a point on the left side of the first square, point I, so that it intersects with the upper right corner of the second square. Later, $EFGH$ was dragged in the direction of line $\overline{IE}$ so that P and Q coincide. Then, the squares are placed in a circle, such that it is the smallest for the squares to fit in. Given that the squares initially share a strip $\sqrt{6}$ m wide, find the area of the region inside the circle and outside $\triangle ICE$. $\left[(26\pi - 24 + 12\sqrt{3})\ \text{m}^2\right]$



14 Let $x, y, z \in \mathbb{Z}$ such that $\gcd(143, 2016) = 143x + 2016y$. Find the greatest value of $|x - y| < 10\,000$. [9406]

15 The keyboard below has 28 keys. A *bullet chord* is a 5-note chord defined as follows.

(a) The 2nd note should be at least 3 notes to the right of the 1st note.

(b) The 3rd note should be at least 4 notes to the right of the 2nd note.

(c) The 4th note should be at least 5 notes to the right of the 3rd note.

(d) The 5th note should be at least 6 notes to the right of the 4th note.

How many distinct bullet chords can be made in this keyboard? [2002]