1 Suppose M, S, and A are real numbers such that $M + S + A = MSA$. Express

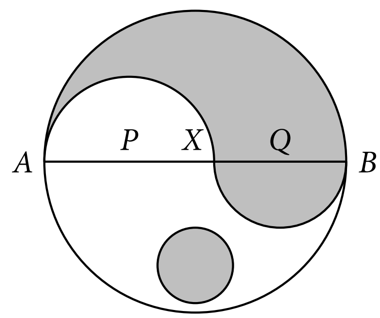
$$M(1 - S^2)(1 - A^2) + S(1 - A^2)(1 - M^2) + A(1 - M^2)(1 - S^2)$$

as a monomial. [$4MSA$]

2 How many numbers are there between 10^{143} and 10^{5254} such that the sum of their digits is 2? [13 794 589]

3 How many numbers less than 2017 are both a sum of two consecutive postive integers and a sum of five consecutive integers? [201]

4 An arbitrary point X is placed in a diameter $\overline{AB}$ with length 160 m of circle O such that it divided the diameter into two unequal parts. Two semicircles with centers P and Q were drawn as shown such that the area of the unshaded area is equal to its shaded counterpart. If the shaded circle inside the unshaded portion of the circle has the smallest possible radius of integral length and every definable segment that AB contains also has integral length, determine the ratio of $|AX| : |XB|$. [$9 : 7$]

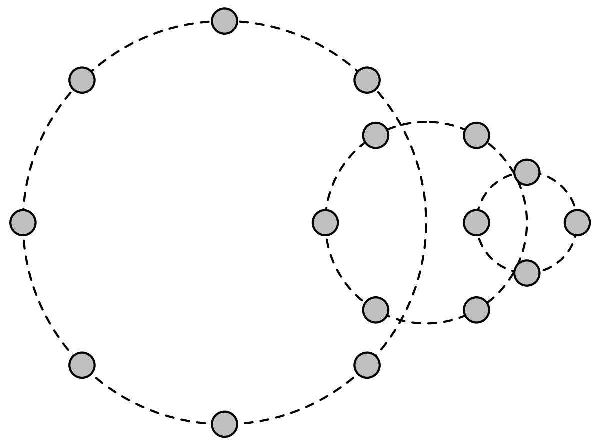


5 Game of the Generals is a 2-player board game composed of 2 teams with 21 soldiers of different ranks (1 flag, 2 spies, 6 privates, 5 generals of different ranks, and the rest are soldiers of different ranks) placed anywhere on the first 3 rows ofthe player's side on a board with 9 columns and 8 rows with respect to the players' sides. Now, the players plan the positions of their soldiers on the board, one of them decides to randomly choose a soldier and rndomly place it on the first 3 rows of the board. What is the probability that the first 5 chosen soldiers are the flag and 4 generals placed on the board where the generals surroud the flag on front, back, and both sides? $\left[\dfrac{1}{2 \cdot 3^5 \cdot 5 \cdot 13 \cdot 17 \cdot 19 \cdot 23}\right]$

6 Let α be a complex zero of $f(x) = x^3 - 2x - 2$ and a, b, c be rational numbers such that $\dfrac{1 + \alpha}{1 + \alpha + \alpha^2} = a + b\alpha + c\alpha^2$. Find $a + b + c$. $\left[\dfrac{2}{3}\right]$

7 Determine a function $f : \mathbb{R} \mapsto \mathbb{R}$ such that $f(x) + f(y) - f(x + y) = xy$ for all $x, y \in \mathbb{R}$ and $f(3) = \dfrac{7}{2}$. $\left[f(x) = \dfrac{8}{3}x - \dfrac{x^2}{2}\right]$

8 Four cans of Red Bull Energy Drink are placed on a circular tray in a circular manner. The tray and five more cans are placed on a larger circular tray, also in a circular manner. As seen in the image, the tray with nine cans and seven more cans are placed similarly on a larger tray. Such manner of placing cans and trays in a larger tray is repeated. If the ratio of th numbers of arrangements for 2018^2 cans to fit into the least iteration to th number of arrangements when the 2018^2 cans are placed in a single circular tray can be expressed as $a^2 : b$, where a and b are relatively prime, find the value of $a + b$.
Note: All the cans of Red Bull Energy Drink are unique. [1010]



9 Suppose there are 6969 players standing around in a circle. They play the following game.

(a) The first player, who was arbitrarily chosen, eliminates the player to the left.

(b) The nearest noneliminated player to the left of the most recently eliminated player eliminates the nearest noneliminated player to the left.

Determine the position of the winner with respect to the first player. *("Position" refers to the number of steps in the clockwise direction.)* [5747]

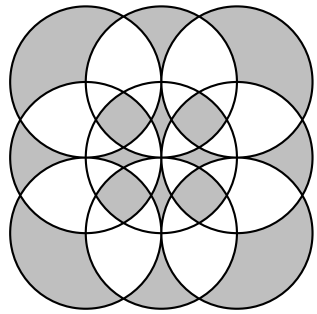
10 Martin, Jhoie, Angela, Rus, and Patrick are all playing a game called Crossing. In the middle of their group lies 4 plates, each containing 1 candy. When Martin says go, all 5 players simultaneously point at a random plate. If two or more players point at the same plate, they don't get the candy. However, if only one player points at a plate, they get all the candies on it. After each round, plates are refilled with 1 candy if they were previously empty, and are increased by 2 if the plate still had candies. What is the probability that Martin has 9 candies after 3 rounds? $\left[\dfrac{3^5 \cdot 5 \cdot 19}{2^{26}}\right]$

11 If a is the greatest power of 2016 that divides 2016! and k is the greatest power of 2017 that divides $(2017^a)!$, find the value of $2016k$. $[2017^{334} - 1]$

12 The centers of three externally tangent circles form a right triangle MSA, whose right angle is located at A. The points of tangency form a smaller triangle such that its side lying inside the circle with center at M is longer than its side lying inside the circle with center at S. If the smaller triangle's perimeter is $(36\sqrt{2}+24)$ cm and $\cos^4 \dfrac{m\angle M}{2} + \cos^4 \dfrac{m\angle A}{2} = \dfrac{137}{100}$, determine the ratio of the smaller triangle's area to triangle MSA's area. $\left[\dfrac{3}{25}\right]$

13 Numbers $1, 2, 3, \ldots, 16$ are arranged in an aray of four rows and four columns such that the product of the entries of the first row is $2^7 \cdot 3^2$; the product of the entries of the second row is $2^2 \cdot 3 \cdot 5^2 \cdot 7$; third row, $2^2 \cdot 3^2 \cdot 7 \cdot 11$; fourth row, $2^4 \cdot 3 \cdot 5 \cdot 13$; first column, $2^8 \cdot 3^2$; second column, $2^8 \cdot 3^2$; third column, $3^4 \cdot 5^2$; and fourth column, $2^7 \cdot 5 \cdot 7$. What is the product of the first entry of the first row, seond entry of the second row, ..., fourth entry of the fourth row? [24]

14 Four circles are drawn such that they are externally tangent to exactly 2 of them and exactly have 4 points of tangency. A 5th circle is drawn such that it passes through the points of tangency. Lastly, another 4 circles are drawn centered at each point of tangency and pass through the center of the 5th circle. If all circls are identical with radii of 1 mm, find the area of the shaded region. $\left[\left(12 - \dfrac{\pi}{3} - 2\sqrt{3}\right) \text{ mm}^2\right]$



15 A knight is given a quest to finish a 7×6 puzzle with S as the start point, E as the end point, and Greek letters δ, ε, θ, λ, ρ, and ω as portals.

S	ε	◊	◊	◊	ρ	θ
△	ε	△	△	△	◊	△
◊	△	◊	△	ε	△	◊
◊	◊	◊	◊	△	◊	δ
δ	△	λ	△	◊	λ	◊
θ	△	△	◊	△	ρ	E

Now, the king said the following rules and conditions:

(a) All tiles must be jumped at least once.

(b) Portals must be used only once.

(c) All jumps must be 2 squares upward or downward and 1 square sidewards; or 2 squares sidewards and 1 square upward or downward (L-shape).

(d) Succeeding jumps can be made n seconds after the previous jump for every inscribed n-gon stepping on.

(e) All polygons except the one he's stepping on are changing (triangles become diamonds and diamonds become triangles) for every time he stepped on another tile.

Also, the knight knows the following:

(a) Each jump takes a second.

(b) Each warp takes 5 seconds.

But, there is still a problem. The king's timer has a defect. Luckily, the time keeper has his own timer. He records every second and compares his timer to the king's timer, as shown on the table.

Time Keeper's Timer	King's Timer	Time Keeper's Timer	King's Timer
00:00:01	00:00:01	00:00:06	00:00:14
00:00:02	00:00:03	00:00:07	00:00:17
00:00:03	00:00:05	00:00:08	00:00:21
00:00:04	00:00:08	00:00:09	00:00:25
00:00:05	00:00:11	...	...

What is the minimum time (in seconds) to finish the puzzle based on the king's timer? [945 seconds]