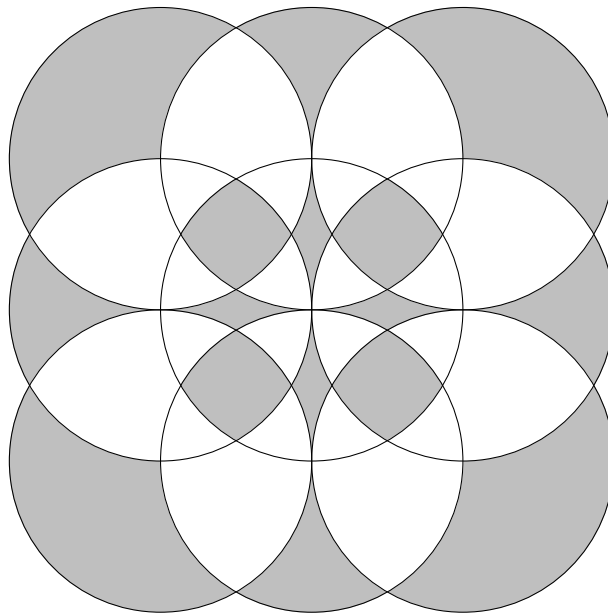


DISCLAIMER: Questions are not verbatim, and are obtained from multiple contestants' retellings of the questions.

- 1 If $M + S + A = MSA$, simplify $\sum_{\text{cyc}} M(1 - S^2)(1 - A^2)$.
- 2 Find all integers x from 10^{143} to 10^{5254} whose digit sum is 2.
- 3 Find all integers x less than 2017 that can be expressed as a sum of two positive integers and a sum of five integers.
- 4 A circle is given with diameter $AB = 160$. X is a point on AB such that $AX \neq XB$, where AX and XB are integers. Another circle with integral radius is drawn such that the area of the shaded regions is equal to half of the total area. Find $\frac{AX}{XB}$ such that the radius of the smaller circle is at a minimum.
- 5 Among 21 game pieces are 5 generals and a flag. One randomly picks 5 pieces and puts them randomly in the cells of a 3×9 board. What is the probability that one ends up with a flag and four generals placed to its north, south, east and west?
- 6 If α is a complex root of $\alpha^3 - 2\alpha - 2 = 0$, and $\frac{1 + \alpha}{1 + \alpha + \alpha^2} = a + b\alpha + c\alpha^2$, find $a + b + c$.
- 7 A function $f : \mathbb{R} \mapsto \mathbb{R}$ satisfies $f(x) = f(y) - f(x + y) = xy$. If $f(3) = \frac{7}{2}$, find a possible value for $f(x)$.
- 8 2018^2 distinguishable Red Bull cans are arranged in the following way: 4 cans are placed on a circular disc. Six cans are on a second disc, and each succeeding disc has 2 more cans than the one before it. Also, every two consecutive discs intersect with one Red Bull can in it. If the ratio of the number of ways to arrange these cans in this fashion to the number of ways to arrange 2018^2 cans in a straight line can be expressed in the form $\frac{a^2}{b}$, where a and b are positive coprime integers, find $a + b$.
- 9 6969 people are seated around a round table. The people, starting from person 1 (to person 2 and so forth) say alternately "One" and "Two". All those who say "Two" are eliminated. This goes on until one person is left. What is the number of this person?
- 10 On a table are four plates, each with one candy on it. Five people aim to get these candies. For each round, the five randomly point at a plate. If a plate is picked by exactly one person, the person gets all the candies on the plate and one candy is placed on the plate. Otherwise, the candies remain and two more candies are placed on the plate. What is the probability that after three rounds, a specific person gets 9 candies?
- 11 If a is the greatest power of 2016 that divides $2016!$ and k is the greatest power of 2017 that divides $(2017^a)!$, find $2016k$.
- 12 The centers of three externally tangent circles are M , S and A , where $\angle MAS = 90^\circ$. Also, $\cos^4\left(\frac{\angle M}{2}\right) + \cos^4\left(\frac{\angle S}{2}\right) = \frac{137}{100}$. The perimeter of the triangle whose vertices are the points of tangency is $36\sqrt{2} + 24$ cm. Find the ratio of the area of this triangle to triangle MSA .
- 13 Integers 1 to 16 are placed in the cells of a 4×4 grid such that products of the integers in the 4 rows are $2^7 3^2$, $2^2 3 \cdot 5^2 7$, $2^2 3^2 7 \cdot 11$, and $2^4 3 \cdot 5 \cdot 13$, respectively; and those of the 4 columns are $7 \cdot 11 \cdot 13$, $2^5 3^2$, $3^4 5^2$, and $2^7 5 \cdot 7$, respectively. Find the product of the numbers in the main diagonal.
- 14 Find the shaded area. The nine circles all have radius 1 mm and adjacent circles are either externally tangent or pass through each others' centers.



15 Some chess problem with portals.