Oral Round

Tier 1

T1-1 *(15s)* A 6×16 rectangle is continuously rotated around its center. Find the area of the new figure.*
$[73\pi]$

T1-2 *(10s)* Find the sum of the last digits of $1^{2017}, 2^{2017}, 3^{2017}, \ldots, 1000^{2017}$. $[4500]$

T1-3 *(30s)* Choko has a set of three numbers forming an arithmetic progression. Kiss has another set of three numbers forming a geometric progression. If I add the corresponding numbers of both progressions, I get 76, 70, and 88. Adding Choko's numbers, I obtain 156. What are Kiss's numbers?
$[6, 18, 54]$

Tier 2

T2-1 *(40s)* Lee Cooper has a $80 \times 80 \times 80$ cube composed of $1 \times 1 \times 1$ cubes. He paints the entire outer surface of the $80 \times 80 \times 80$ cube and then removes these painted cubes. He repeats this process until he can no longer remove any more cubes. If he gets a random $1 \times 1 \times 1$ cube, what is the probability that the cube is painted on at least 2 faces? $\left[\frac{119}{3200}\right]$

T2-2 *(30s)* Roma, a mathematician, asked her son, Maro, to unlock a safe for her. She said the password p is a prime number such that $32p + 1$ is equal to the cube of a positive integer. What is the password?
$[1123]$

T2-3 *(15s)* Let M, S, and A be the roots of the polynomial $f(x) = 127x^3 + 1729x + 8128$. Find $(M + S)^3 + (S + A)^3 + (M + A)^3$. $[192]$

Tier 3

T3-1 *(20s)* If there are 6048 ways to seat any r people out of n around a circular table having r chairs, how many ways can you seat any r people out of n in a row of r chairs? $[30\,240]$

T3-2 *(30s)* If $x + y = xy = 5$, find the value of $x^5 + y^5$. $[625]$

T3-3 *(20s)* The smallest integer greater than or equal to the nth root of n is denoted as $\lceil \sqrt[n]{n} \rceil$. What is the product of all $\lceil \sqrt[n]{n} \rceil$ for n from 1 to 2017? $[2^{2016}]$

Tier 3 Clincher

T3-C1 *(10s)* If five-digit codes are formed using the digits from the decimal number system, what is the probability that the code formed when reversed (i.e. the first digit is switched with the last and the second is switched with the fourth) yields the same code? (e.g. 0-0-0-0, 0-0-1-0-0, etc) $\left[\frac{1}{100}\right]$

T3-C2 *(15s)* Given $\log 2 \approx 0.3010$, $\log 3 \approx 0.4771$, and $\log 7 \approx 0.8451$. How any digits does 2016^{2016} have?
$[6662]$

T3-DoD *(2min)* A container holds 2500 ml of chocolate drink. Every time Joze eats breakfast, he'd get a glass of the chocolate drink and for some reason he'd replace it with the same amount and same chocolate drink. One day he ran out of chocolate drink so he replaced whatever he drank with milk instead. If the capacity of the glass is 250 ml, what percent of the original chocolate drink would still be present in the container after 5 days? $\left[88\frac{8}{9}\%\right]$

* Voided because of ambiguity.

Tier 4

T4-1 *(30s)* Uber driver Lee Cooper draws thirteen cards from a standard card deck. For each jack drawn, he assigns one point; for each queen, two points; for each king, three points; and for each ace, four points. In how many different ways can he have a hand worth six points? [152]

T4-2 *(25s)* Maro, Mario, and Luigi have the same birthday. Today, Maro is 3, Mario is 15, and Luigi is 21. Luigi's age will be x when his age is equal to the sum of the ages of Mario and Maro. A certain positive integer has exactly 16 factors such that two of its factors are 21 and x. What is the sum of these 16 factors? [480]

T4-3 *(15s)* Nika got the highest score in the first quiz with a score of 9 out of 10. Everyone else got a low score, so the professor decided not to record the quiz but gave Nika the option whether to record hers or not. If at the end of the semester the total number of points for the quizzes is 2017 (excluding the first quiz), what is the minimum integer score of Nika out of 2017 so that the option not to record her score in the first quiz would result to a higher percentage in her grade for quizzes? [1816]

Tier 5

T5-1 *(25s)* A regular dodecagon has area $72 + 36\sqrt{3}\,\text{m}^2$. Determine its perimeter. $\left[24\sqrt{3}\,\text{m}\right]$

T5-2 *(30s)* How many sets of three distinct single-digit numbers may be formed such that their product is a product of powers of exactly three prime factors? [29]

T5-3 *(25s)* A friendship of three people consists of one person known by two other people who do not know each other. A group of people satisfies the following conditions: Only one person is on exactly 3 friendships of three. The rest are on only 2 friendships of three. If the number of people in this group is the least that satisfies the said conditions, how many friendships are there in the group? [3]

Tier 6

T6-1 *(45s)* If the thousands digit of a 4-digit perfect square number is increased by 3, its hundreds digit decreased by 2, and its units digit increased by 8, a new 4-digit number is formed. What is the sum of all possible solutions for the original number? [6442]

T6-2 *(25s)* How many $(a, b) \in \mathbb{Z}^2$ such that the nonzero complex number $\dfrac{\sqrt{ab + 2017}}{ab + 100} - \dfrac{\sqrt{|a + b|}}{ab + 100}i$ is real? [87]

T6-3 *(20s)* Maro wanted to visit his dad, Mario, at his workplace via Uber. He called him and asked how far their house is from his workplace. Mario said that it is the smallest integer greater than $\left(\sqrt{10} + \sqrt{5}\right)^6$ meters. How far is their house from Mario's workplace? [24 750 m]

Tier 7

T7-1 *(15s)* Maro, Arbee, Brian, and Tracy were at a fun fair and they had to guess the number of cans of Red Bull Energy Drink in a large container. Prizes are awarded on how close the guesses were. Luckily, the first prize went to Maro, who guessed 169 cans. The second prize went to Arbee, who guessed 144 cans; the third prize went to Brian, who guessed 121 cans; and the fourth prize went to Tracy, who guessed 194 cans. How many cans were in the large container? [157]

T7-2 *(30s)* Let (a_n) be an infinite geometric sequence with first term a_0 and common ratio r. The sum of the first 2017 terms of (a_n) divided by the infinite sum of the terms of (a_n) is $\dfrac{3}{5}$. Let (b_n) be an infinite geometric sequence where $b_n = a_{n+1}$. for all n. If the infinite sum of b_n is 115, what is the sum of its first 2017 terms. [69]

T7-3 *(20s)* Determine the largest positive integer which is divisible by all integers less than its square root. [24]

Tier 8

T8-1 *(15s)* We call an integer *très belle* if its units digit is one more than twice the sum of its other digits. For example, 2017 is très belle since $2(2+0+1)+1 = 7$. How many integers less than 2017 but greater than 1000 are très belle? [11]

T8-2 *(20s)* In the early 70s, Ferdenend had x gold bars in his stash. He kept $\frac{2}{3}$ of them for himself. Then, he divided the remaining gold bars and gave equal amounts to Emelde, Bengbeng, and Emee but 2 gold bars were left. He included Eymee on the sharing but 3 gold bars were left. Now, in order to equally divide the remaining gold bars with no remainder, he included his 2 bodyguards on the sharing. Find the smallest possible value of x. [357]

T8-3 *(30s)* Suppose $\overline{THEAB} + \overline{IROEN} = \overline{MINM4N}$,where each letter represents a unique 1-digit non-negative integer. In how many ways can this be true? [4]

Tier 9

T9-1 *(30s)* Given $M^2(S+2A)+S^2(A+2M)+A^2(M+2S) = 10$, and M, S, and A are positive real numbers, find the maximum value of MSA $\left[\frac{10}{9}\right]$

T9-2 *(20s)* If $x^{14} + \frac{1}{x^{14}} = 47$, and $y^{18} + \frac{1}{y^{18}} = 4$, what are the last 2 digits of $x^{3584} + \frac{1}{x^{3584}} + y^{4608} + \frac{1}{y^{4608}}$? [41]

T9-3 *(45s)* There is a video on YouTube called "The Entire Bubuyog Movie but every time they say 'bubuyog', it gets faster." In particular, the movie speeds up by 1.25× its previous speed. When the sped up video is played, the word 'bubuyog' is mentioned at the following time arks: 00:17, 1:05, 4:17, X, and 20:17. If the video clip lasts only 37:21, and the original movie lasted 1 hour, 31 minutes and 42 seconds, determine the time mark X. Round off answers to the nearest second. [16:01]

Tier 10

T10-1 *(40s)* A normal year in SHINee World is composed of 364 days, but unlike Earth, it has 5 "days" a week, namely Onew, Jonghyun, Key, Minho, and Taemin, with Onew being the first day and Taemin being the last day of the week. In this alternate universe, a leap year does not exist; instead, it has kick and hop years. A kick year occurs every 3 years an has 367 days, a hop year occurs every 2 years and has 363 days, and every 6 years is only a normal year. If the year $XYXX$ ends on an Onew, what nearest year starts on a Taemin? $[XYXX - 1]$

T10-2 *(20s)* Lee Cooper, a fashion and numbers enthusiast, likes labeling numbers that have a certain number of factors. He calls numbers that have exactly 3 positive divisors *fab* while the ones with exactly 9 factors *glam*. How many fab numbers and glam numbers are under 3000? [31]

T10-3 *(30s)* Rus and Rica, both programmers, just had their firstborn. However, they can't decide on the name of their eldest. So they made a program that will print all the possible 4-letter combinations of the English alphabet. The letter combinations follow the format of a counter where AAAA is equivalent to (0000), and is the first combination, followed by AAAB, which is equivalent to (0001), then by AAAC, equivalent to (0002). However, in this case, AAAJ (0009) is **not** followed by AABA (0010) but instead, is followed by AAAK (000[10]), and then followed by AAAL (000[11]) until AAAZ (000[25]). After AAAZ is AABA, equivalent to (0010), followed by AABB (0011), until AABZ (001[25]). Next to AABZ is AACA (0020) and so on and so forth until ZZZZ ([25][25][25][25]). Now, how many combinations were printed from AAAA to UBER? There is one letter combination from AAAA to AAAA and there are two letter combinations from AAAA to AAAB. [352 318]

Tier 11

T11-1 *(45s)* Find the total area of the region outside the unit circle and a smaller circle tangent to the unit circle but inside the square with vertices $(0,0)$, $(0,1)$, $(1,0)$, and $(1,1)$ such that the diameter of the smaller circle with $(1,1)$ as an endpoint is parallel to the x-axis. $\left[\dfrac{32-9\pi}{32}\right]$

T11-2 *(30s)* Riding an UberX, it takes $(n+1)^6+(n-1)^6+5$ kilometers from the first stoplight Maro will pass by for him to arrive at Mario's workplace. If n is a positive odd integer, and for every 8 kilometers there is a stoplight, how far is Mario's workplace from the last stoplight that Maro will pass by? [5 km]

T11-3 *(25s)* A tetrahedron is a 3-dimensional figure with 4 triangular faces, 6 edges, and 4 vertices. Consider an irregular tetrahedron with two opposite edges $\overline{AB}$ and $\overline{CD}$ having the same length 6. A line with length 4 connecting the midpoints of these edges is perpendicular to the edges. The lines connecting the midpoints of $\overline{AB}$ to C and D are also perpendicular to $\overline{AB}$. Find the volume of the tetrahedron. [24]

Tier 12

T12-1 *(45s)* I draw a series of line segments such that B, the second endpoint of a line segment, $\overline{AB}$, is the first endpoint of the next line segment, $\overline{BC}$, and $m\angle ABC = 120^\circ$. $\overline{AB}$ is the first line segment, $\overline{BC}$ is the second, $\overline{CD}$, is the third, $\overline{DE}$ is the fourth and so on, with $m\angle ABC$, $m\angle BCD$, and $m\angle CDE$ all equal to 120° and $\triangle ABC$ overlaps with $\triangle BCD$, $\triangle BCD$ overlaps with $\triangle CDE$ and so on. If every line segment is half of the length of the previous one, the series of line segments eventually becomes a single point X. What is the ratio of $|\overline{AX}|$ to $|\overline{AB}|$? $[\sqrt{2}:1]$

T12-2 *(20s)* Simon and Ricardo were tasked to deliver 2017 cans of Red Bull Energy Drink. Along the way, there are sets of three thirsty trolls who takes these energy drinks in order.

- The first troll takes 1 from each person who carries an even number of cans and takes 2 from each person who carries an odd number of cans.
- The second troll takes 3 from each person who carries an even number of cans and takes 4 from each person who carries an odd number of cans.
- The first troll takes 3 from each person who carries an even number of cans and takes 0 from each person who carries an odd number of cans.

Before each roll, they decide how many cans each of them are carrying but each person must carry at least 4. If there are 69 sets of trolls in their way, what is the maximum number of cans Simon and Ricardo can bing to their destination? [1464]

T12-3 *(40s)* Twin primes are primes that differ by 2. Sexy primes are primes that differ by 6. Let $m < 6969$ be a product of twin primes p and q, with q the smaller prime. Let $n \neq m$ be the resultant palindromic number when the hundreds and ten digits of m are swapped, which can be written as product of 2 other primes r and s, with s the smaller prime. If q and s are sexy primes where $q > s$ and p and r are the smallest and largest primes respectively of a sexy prime triplet, find q. [29]

Final Round

Wave 1

W1-1 Let M be a positive integer whose digits' sum is 2017, and let a, b, c, d, e, and f be distinct single digits. Given that $a + b = d$, $b + c = e$, and $d + e = f$, with a equal to the sum of the digits of $M + 1$, find all possible solutions for the values of a, b, c, d, e, and f, in ordered sextuples.
$[(2,1,4,3,5,8),(2,1,5,3,6,9),(2,3,1,5,4,9)]$

W1-2 A positive integer n^5 has 2016 (unique) divisors and it is divisible by 29. If the sum of the prime divisors' exponents, upon prime factorization of n^5, is between 123 and 143, determine the prime factorization of the least possible n. $[2^{25} \cdot 29^3]$

W1-3 A palindromic number is a number that remains the same when its digits are reversed. For example, 1991, 525 and 6 are palindromic numbers. Find the probability that a positive number less than 10^{2017} is a palindrome. $\left[\dfrac{11 \cdot 10^{1008} - 2}{10^{2017} - 1}\right]$

Wave 2

W2-1 Define an Uber-romantic number to be a positive integer such that if all of its digits except those that are equal to 1, 3 and 4 are removed, the remaining digits (in their original order) will for the number 143. For example:

- 1 928 473 is an Uber-romantic number.
- 214 902 131 is not an Uber-romantic number since 14 131 is not equal to 143.
- 51 234 509 is not an Uber-romantic number since 134 is not equal to 143.

How many 7-digit Uber-romantic numbers are there? [77 175]

W2-2 For a party of 5, there are 44 ways to give one gift to another person such that each person can only receive one gift. How many such ways are there for a party of 6? [265]

W2-3 On a 2×3 grid, movement from one vertex to another is restricted along the lines. How many paths of length seven are there from the lower left corner to the upper right corner? [189]

Wave 3

W3-1 For a party of 6, there are 265 ways to give one gift to another person such that each person can only receive one gift. Of these 265 'ways', what is the minimum number of 'ways' we can choose so that we can ensure that out of all these 'ways' everyone has given a gift to every person? [213]

W3-2 The *Euler totient function* of n, denoted as $\phi(n)$, gives the number of positive integers less than or equal to n, that are relatively prime to n. Find the smallest positive integer m such that $\underbrace{(\phi \circ \phi \circ \cdots \circ \phi \circ \phi)}_{m \text{ times}}(29^{2017}) = 1$. [8069]

W3-3 A fair 6-sided die is rolled 10 times. For the ith roll, the face value V_i is recorded and is given a score S_i of -1 if the value was odd, and a score of 1 if the value was even. After 10 rolls, the scores are added to make a total score. Determine the sum of all total scores among all the different ways that the sum of values $\sum_{i=1}^{10} V_i = 17$. [−42 720]

Wave 4

W4-1 Maro and his friend Abe play a 'sweets game'. There are 29 sweets on the table, and they must take turns eating as many sweets as they choose, but they must eat at least one, and never more than half of what's left. The loser is the one who has no valid move. If Abe has the first turn, what must be the sequence of the number of sweets that she will leave during her turn in order to win with optimal play (least number of turns)? [15, 7, 3 and 1]

W4-2 Evaluate: $\frac{2}{3} + \frac{1}{2} + \frac{2}{15} + \frac{1}{12} + \frac{2}{35} + \cdots + \frac{1}{264} + \frac{2}{575}$. $\left[\frac{851}{600}\right]$

W4-3 Let x, y, and z be numbers such that $x = y^3 + z^3$, $y = z^3 + x^3$, $z = x^3 + y^3$, S_1 be the set of real solutions to the system, and S_2 the set of complex solutions with distinct components. Find the sum of the number of elements of S_1 and the number of elements of S_2. [9]

Wave 5

W5-1 Patrick and Rus are playing a game called 7 Blunders. Each player starts with 7 cards in their hand with the following scores labeled for each card: 1, 3, 4, 4, 4, 5, 7. To play, both players will simultaneously draft a random card from their hand to keep, and swap the rest with the other player. They do this repeatedly until both players have drafted 7 cards. They then total up the score.

What is the probability that Rus has a higher score than Patrick, and therefore wins? $\left[\frac{36}{79}\right]$

W5-2 A right circular cone of radius 42 m has volume V_0. A frustum of volume V_1 is placed below the cone such that it forms a bigger cone. A second frustum of volume V_2 is added to the new cone to form a bigger one, and so on. If $V_n = \frac{279}{343} \cdot V_{n-1}$ such that n is a positive integer, to what value does the radius of the entire cone approach as $n \to \infty$? [73.5 m]

W5-3 Find all ordered triples (M, S, A) which satisfies the equality $(M^2S + MS^2)^A = (\sqrt{M} + \sqrt{S})^4$. [(8, 8, 1), (1, 1, 4)]