

MATHIRA 2018 REFLECTIONS

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Figure 1: A picture at the photobooth.

This is probably my last competition report for math, unless I make IMO, which I doubt I will. (And um, I'm kind of getting tired of writing these. Don't tell anyone.)

1 OVERVIEW

This year is the 30th year of Mathirang Mathibay, hooray for round numbers. I just realized that this makes Mathira around as old as PMO. Anyway, the theme this year was Spectrum, symbolized by a hexagon with a line through it, in the colors of... a spectrum. Representing the "whole spectrum" of math that Mathira wants to represent, supposedly.

Any high school can send up to two teams of three participants with an optional alternate. There is an elimination round in the morning and an oral round in the afternoon, of which the most accurate description is a

I used to claim that MMC had the largest prize money, followed by Mathira. Now that I think about it, PMO has larger prize money in aggregate.

cutting board: two teams are eliminated each tier of the oral round, from twenty-five, ending with four. The other things Mathira is known for is absurdly fast time limits and sizable prize money, which both appeared again, as tradition.

2 PRE-COMPETITION

This is the fourth year I'll be joining Mathira. Three years ago I was with Axi and Nikki or Kelly or Louie I think, and it was at UP Arki. The year after that the mechanics changed, and since then it's been at NISMED, including this year.

Mathira is a bit more later this year, on February 26. It seems to be shifting later and later – it used to be January, and then last year it was February 13, and now it's all the way to February 26. Eventually it will cycle and they'll skip a year or something.

They already did in 2011 I think.

We weren't able to prepare at all. We've been pretty busy preparing for the MMC. Well, *they* were busy, I was helping coach them. We were just by ma'am (Marilyn) Soriano during the sectoral orals that Vincent (Dela Cruz), (Vincent) Carabbay and I were going to join. And again, we only had enough money to send one team in, so yeah:

Valenzuela City School of Mathematics and Science (VCSMS), Team A:

- Carl Joshua Quines (me), grade 12.
- Mark Vincent Carabbay (Carabbay), grade 10.
- Vincent Dela Cruz (Vincent), grade 9.

We were informed to be there at around 6:15 AM, roughly the time of the flag ceremony, because we were only going to commute. So yeah.

3 MORNING

I was at school late for the flag ceremony, at about 6:20 AM or so, so the guard wouldn't let me in. Then sir (Romeo) Miclat I think saw me through the gate and made a lot of gestures. Then ma'am (Regina) Gomas and Vincent came by, and they pulled me through the gate.

They haven't seen Carabbay yet. We waited for quite some time, until about 6:30 or so, and Carabbay still hasn't arrived, and ma'am Gomas was starting to worry. This was usual, we explained, Carabbay was almost always late for contests.

And then ma'am Soriano came and she was like, "Carabbay isn't here?!" and then she started to go on about how we're going to be late, and the

round was going to start at 8 AM or so, and we're only going to commute, and stuff like that.

The flag ceremony ended not long after that, and then Carabbay all of a sudden steps out of line and goes to us. It turns out he was lined up in the flag ceremony all along. And then kuya JR (Roxas) was also there, and it turns out he could drive us there instead of us commuting, which was awesome.

It was not a long drive, because kuya JR took NLEX and then Mindanao Ave to get there. Soon enough, we were again at IM, the University of the Philippine's Institute of Mathematics.

At IM, we initially tried to go through the side entrance. But then we were blocked by one of the students, who told us to go through the front entrance instead for registration. The front entrance involved a lot of climbing, and by the time we reached the top, we paid for our registration.

We said hi to the Xavier team, who was as early as we were. Mentioning Ryan (Shao) and Philmon (Wee) – I'm sorry to say I forgot the third member.

huhu pls forgib me i am bad with names



Figure 2: Ma'am Gomas and the others.

As was Mathira for the past year or two, we took a team picture. We tried to pull off the Valenzuela City School of Makati thing once again, after the recent MMC gave some new names for our school: Valenzuela City School, ValMaSci National High School, and (my favorite), Valenzuela. Just Valenzuela. But there were only serious pictures.

Since this was my last year competing, I decided to buy a shirt, after buying the Sipnayan shirt this year as well. This year's shirt was black,

*The Metrobank-
MTAP-DepEd
Mathematics
Competition.*

with the Spectrum logo in front. I asked Carabbay about this, and he said it the red shirt was more *bagay*, since I wasn't "fat enough to wear black". But when ma'am Gomas bought a shirt and I tried it on, Carabbay changed his mind, and insulted my waist circumference while he was at it. So I bought it.

Ma'am Gomas had breakfast for us that she cooked herself, which was really nice of her. So we ate it at the canteen. Seeing the 1M canteen reminds me of MOSC days... and I found myself missing the boxed rice meals. Vincent agrees, and ma'am Gomas says she'll buy that for us for lunch later.

I think I got most of the last names out of the way.

After breakfast, which took quite some time, we go up again and then talk to people! I gave everyone I saw a hug to greet them. There was Andres (Gonzales), and I gave him a hug, and Steven (Reyes) and Stefan (Ong), who were both huggable. I saw Bryce (Sanchez) and Shaq (Que) for GCC (Grace Christian College) and Shaq was not huggable, so I was not able to hug Bryce. I also said hi to Jinger (Chong), also on the St. Jude (Catholic School) team. And Vince (Torres), and (Immanuel) Balete.

Then the opening ceremonies began outside 1M, and the sun was quite hot. We all had our umbrellas out as we lined up per room number. The problem was, there were people who held up signs for rooms 301, 302, 303, and 304, but no 306. A few minutes later, someone came out holding a 305 sign, but still no 306.

The chronology here is very likely wrong. But they did come later than expected.

And then Pisay (Philippine Science HS) came: Kyle (Dulay) came up to me. I remarked that I thought they wouldn't come, which would have made for an easy win, and promptly asked them to leave. He replied: no.

A few minutes later, while we were burning in the heat of the sun, the guy holding the 306 sign appears! So we line up. And shortly after, the opening ceremonies start with a national anthem, followed by a couple of short speeches. I think the memorable part was by the faculty president, who said that the problems intended to show the full spectrum of mathematics – "algebra, statistics, geometry, and trigonometry". I am quite sure there was no number theory or combinatorics mentioned.

4 ELIMINATIONS

We were lead into our rooms, and the mechanics were explained, and we were given time to go to the restroom. And then all of a sudden, the DLSU team appears, with Luke (Bernardo) and Albert (Patupat) – what a surprise! I was completely expecting them to not go, because of issues with participation or something. But they were there, for better or for worse. Also in our room was the GCC team.



Figure 3: Our room during the eliminations.

I tell them who else are here, and then I ask if they've seen Steven (Wang). We were going to all take a picture together, us graduating seniors. And they said they have, so yes.

The mechanics, briefly: there are fifteen questions. The n th question is worth n points maximum and is for n minutes. After the n minutes, it is still possible to submit answers to the question, but the points it is worth decays by some function. Contestants are informed whether their answers are correct or incorrect, and can submit as many times as desired.

We again make our plan to answer 69 to any question with a reasonable answer of 69, but it would not fare well. Some highlights from the round:

1. Once again, we do not answer the first question in time. I got it two minutes after start of round with some trial-and-error: the non-trivial pair was $(4, 2)$.
2. Late, again. This is not a double counting problem, this is a construction problem: one such construction is a 3×3 tiling, with a 2×2 box of maroon squares and an L-shaped block of green squares. I ask Carabbay to help with arithmetic.
3. We would end up not answering this question long after the round ends, because it involved an ugly quartic we couldn't reduce. It turns out no one we asked solve this question either. We eventually give up on this problem some time in the middle of the round because it has decayed too much.
4. Vincent got this on his own while Carabbay and I were working on 3.

It was pointed out this could lead to trial-and-error. A person on UPMMC remarked this was not a bug, but a feature.

5. Carabbay got this on his own while Vincent and I were working on 3.
6. Vincent and I tried this problem as Carabbay rejected it, working on 3 instead. We tried to find a telescope. I would work on this problem later on and did find a telescope, but couldn't get the answer, even after trying multiple times.
7. Vincent and Carabbay worked on this while I worked on 6. I remember that they solved it.
8. Vincent worked out the first, nice, integral tuple, and the value of x for the second tuple. Carabbay bashed out the whole triple after Vincent couldn't get the arithmetic right. I was still working on 6.
9. I initially worked on this problem after giving up on 6. I was able to translate the condition, but I made a crucial mistake I only realized after the contest. I thought it was the numbers divisible by 2, 3, 5, or 7. But 2 should not be part, whoops. Carabbay tried to help me bash, but we gave up because we thought there were four sets.
10. I worked on this on my own initially, and got a mistake somewhere because I thought there were 100 instead of 101 balls. I worked it out again with Vincent and we got it together, he helped me out with arithmetic. Carabbay was trying 9 at this point.
11. We were all working together at this point, since we've given up on 3, 6, and 9. The main idea of "consider the center" came to us, and Vincent managed to complete the logic to get the answer. We then went back to the other problems, but we were starting to get tired.
12. This... is an interesting problem. We got the answer of $\frac{1}{7}$ in several different ways: geometrically, algebraically, with trigonometry, and so on. But it refused to be accepted. We tried if it were complex numbers, with an answer of infinity, also rejected. We tried if it was zero, if it was a minimum typo, also rejected. We got frustrated by this.
13. When this problem came, we guessed a bunch of answers, but none of them were correct. So they went back to 12. I tried to use the angle formulas as well as complex numbers, with the assistance of Vincent, but at the end we just had this weird fraction that we couldn't reduce.
14. I was working on 13 while Vincent and Carabbay got a hold of this and answered it pretty quickly. I didn't even read the problem.

15. We did not bother answering this problem. By this point, we were pretty resigned to our answers – not having solved 3, 6, 9, 12, 13, and 15. That meant we solved nine problems, and Carabbay was surprised we solved that many. Vincent was surprised too.

Immediately after the round ended, the topic of discussion was problem 12. Turns out the other teams got an answer of $\frac{1}{7}$ and had it rejected as well, so it was proposed to launch a complaint. Certificates were given out and lunch break was called.

5 LUNCH

Albert, Luke, Shaq and I were outside talking about the problems. A bunch more people joined our circle, so many that I forget who precisely were in the circle. It seems that most people solved the same problems. Luke and Albert and Shaq and Kyle share that they correctly guessed $\sqrt{7}$ for problem 13.

It seems that everyone was in agreement that problem 12 was mistaken, and that we needed to complain. So we did – we went to the room where we were supposed to complain, wrote out some solutions, waited for people to come. It was about five minutes in and the problem proposer was still out, and everyone else gave up, so the only people left in the room were just me and Kyle and Andres.

Eventually, after around thirty minutes or so, the original proposer came and we made our case. But then – there was no idea what to do. It would not be fair to void the question, since there was one team there who wanted the points. It was also not fair to just award the points to the teams who answered correctly, since it wasn't known who did, and in what times.

It was eventually decided to just void the question, even though it wasn't completely fair to everyone involved, since we were all hungry and wanted to eat. Hopefully this will get fixed in the future.

I go down and see (Nathanael) Balete-sempai, and we talk about the problems, and about problem 12. He points out that this is one of the disadvantages of having the eliminations round as it is, and he also talked about why he disliked it. I guess we have different opinions – I actually think it's fun, and it's fair in a great number of cases.

We eat lunch in the UPIM cafeteria again. I really missed the place! And the rice and meat you got in these cups, that was awesome. We had that for lunch, as ma'am Gomas bought us some. They were already eating theirs when I started, because I was a bit delayed from complaining about problem 12.

This was unprecedented, and Mathira didn't have mechanisms in place for complaining, apparently.

We spend the time eating and talking about the problems. Vincent opens Wolfram Alpha and tries the last problem. He's surprised not only by the presence of π in the answer, but also a $\sqrt{6}$ I think. This is pretty surprising, because it meant it was more than just the series of squares or something.

After lunch, we were at the ground of UPIM using the WiFi. Balete-sempai was there and we talked about NISMED, and his plight of having to rush there between classes. The St. Jude team was there and we talked about the problems.

We walked to NISMED, and I still kinda remember the path since we've done this walk several times now. You'd take the path that cuts through some weird forest, end up outside the college of science, and then just walk to NISMED.

Now, Carabbay noticed there was a team that was behind us, and he wanted to see if they were just following us. So we intentionally missed NISMED and overshot. Then they took a left turn and walked into NISMED, and then Carabbay was like, oops, they weren't. Ma'am Gomas pointed out that they gave out directions to NISMED anyway.

We went up and there was no Red Bull! The doors to the auditorium were still closed, so we just stayed outside. Balete said that it was possible to still get the air conditioning if you just hovered outside the door, so Vincent and I did that but *nahiya kami* so we went out again.

Red Bull was a sponsor last year, so they gave out free Red Bull to all contestants. The lack of Red Bull this year is sad.



Figure 4: Observe the distinct lack of Red Bull.

Eventually the doors were opened, so we went up and took our seats. There was a lot of talking, of course. Balete-sempai collects the questions from the elimination round from our own recollection. We were all crowd-

Questions posted on <https://cjquines.com/contests> with his permission.

ing him and it was all pretty noisy since everyone was reciting the question at the same time, and it was clear that he was getting confused. It was cute.

There was me, Luke, Albert, Steven, Kyle, Vince. And we all took a picture together, as promised. But Matthew (Isidro) wasn't there – and it was his birthday to boot. Whoops.

And we talked about the usual things, the lots and lots of usual things, and we didn't run out of things to talk about: college apps, MTG and AMSLI, the problems, a dozen other things. It was all good, it was all fun.

6 SEMIFINALS

A brief summary of the rules. There are twelve tiers of three questions each. At the end of each tier, beginning tier 3, the two lowest-scoring teams are eliminated, clinchers if necessary. Later questions are worth much more than earlier questions, see the appendix.

There was the typical opening ceremonies, speeches, introduction of judges, and we were all still talking. When the top twenty-five teams were being announced, we were still talking. In fact, Albert was unaware who the top twenty-five were. The teams were the usual, and then we all went down and sat in our chairs.

Unlike last year, the team number issue was solved by asking teams to write their numbers on the back. Great job, the people who thought of that! The answer was initially to be written on the front, but the judges found this tedious and asked teams to write on the back instead, which made more sense as well.

Let's list the tiers:

1. No one got the first question if I remember correctly, but we were close! We clutched the second question in time, and around a quarter of the teams got it. A couple of teams got the third question. We almost got it as well, but we just ran out of time after the division.
2. We very nearly got the first question but miscounted the number of weeks. We answered 21 instead of 28. A number of teams got this question. The next question, no one got. Then we clutched the third question: at first, we ignored the lower half, getting an answer of $\frac{5}{8}$, but managed to correct it. It seems that Xavier was on a roll today, since they've gotten every question a positive number of teams got so far.
3. We managed to clutch the first question: I was drawing, and apparently Vincent didn't understand the question, so I drew it for him. Then we counted, and they did the arithmetic like one second

The issue of writing team numbers is well-known. The solution is well-known as well: get people to write numbers before the question is read, or be PMO and print the numbers beforehand.



Figure 5: The teams!

before time. We nearly got the second question, but we missolved the quadratic; a lot of people got that question. No one gets the third question.

The second question was funny, by the way. We made a meme of it in the Philippine Meme Olympiad page. It was a good pun.

[https://fb.com/
philmeoly](https://fb.com/philmeoly)

Teams would started to be eliminated this tier, with three teams out. There were several teams who haven't scored yet, so a clincher was held. No one got the first two clinchers, so a do-or-die had to be given. The judges hotfixed the do-or-die rules so that the question was a normal two-minute question. More than three teams were eliminated I think, so no teams would be eliminated for a couple tiers.

4. The first question here is fun. The initial answer was 105, so we raised our protest sign, which was in the shape of the logo this year. A bunch of other teams did as well, so we all stood up and went to the judges to stage a protest. Apparently this was the first time Albert would complain, ever, so we let him do the honors of explaining the solution.

It turns out one of the judges proposed the problem himself, and asked the answer to be changed to 102 beforehand, but it didn't. So that was resolved. The second question was a guess-and-check question: you wanted it to be integers, and since the sum is 3, the only possibility were the radicals being 1 and 2 in some order. (You could quickly rule out 3 and 0.) So both work, and then you take the product. Clutch.

The third question is similar to HMMT 2012 Combinatorics. I would know, because I copied that question for a mock test. It was not a hard question in either case and several teams got it.

5. Only Xavier got the first question here – they’re on fire today! No one gets the next two questions. Vincent resolves to memorize the third question, since it came up a lot.
6. I recognized the first question as the packing fraction of the hexagonal cubic sphere packing or something, and no one got it I think. No one got the next two questions as well. The judges were struggling to come up with more and more different ways to say “no one got the question”, like “The number of people who got this question is zero” or “*Walang nakapagbigay ng tamang sagot.*”



Figure 6: Tier 6/1.

At the end of the tier was a clincher of about five teams I think. No one got the first question, a team got the second question, then a do-or-die was given. At this point, the remaining teams were pretty good, and I was getting scared we might get eliminated soon. . . maybe even in Tier 11, like last year.

I wanted an intermission like last year. But there was no intermission, just a swap of the quizmasters. Sad, but I suppose it was because we were all rushing.

7. The first question is balls-and-urns; a similar question appeared in PMO areas. The second question got me a chuckle: “hEeEey jHemERlyN!!!!” comes to mind. However, it was a bit unclear. For

This is a reference to IV of Spades’s Hey Barbara.

example, was 25 appended at the beginning or the end? How many consecutive integers? And so on.

The original answer of 7 was contested – some teams claimed 8. But then the judges talked among themselves and got 10, so that was that. No team got the correct answer. The same held for the next question, which was very complicated.

Another clincher was held. We tried the problems even though we weren't part of the clincher, and it seems that the second problem was wrong. In either case, no team got any of the questions if I remember correctly.

8. No one gets the first two problems – the first was confusing, the second could have been an oly problem. The third problem is refreshingly doable, except we ran a bit out of time.
9. The first problem is bust. St. Stephen gets the second problem, and I remark that Balete has it in his genes. Balete–sempai's statistics skills passed on to him. Two teams got the third question, DLSU of which is one. I begin to worry, as we haven't gotten a question recently.
10. We get the first question, as well as a bunch of other teams: hooray for linearity of expectation! No one gets the next two questions again. More and more creative ways of saying "no team got the correct answer".
11. No team gets the first two questions. I remember a team getting the last question, and we almost did, but ran a bit short on time. I begin to get scared, but Carabbay reminds us that we have a good lead, and as long as we don't mess up the next tier, we'd be making finals.
12. We did not mess up – no one gets any of the questions. That meant we were in finals, which was nice.

We had to say goodbye to a lot of friends. At the end, there was us, St. Jude, GCC, and Pisay. There were a few brief moments in between the semifinals and the finals, and I spoke to Balete–sempai about some of the problems. The quizmasters read the sponsors for the dozenth time or so.

7 FINALS

I felt happy that we made it to the finals, but exhausted. This was, after all, my last competition in my high school life, and it would be nice to end by doing something no ValMaSci team had done before: make finals!

*Call me greedy, but
money was a big
motivation to win
Mathira.*



Figure 7: The setup in the finals.

I was happy, and winning would be merely a cherry on top, but much appreciated because of the money involved.

The finals was a bit of a blur. The mechanics were like eliminations: you had several tries to answer questions, and you could answer any time, and you would be told whether you were correct or not. Each wave had three questions, which decay at different rates. There were ten attempts over all questions, question value decayed over time, over six minutes.

Carry over scores were revealed: we and GCC had 20 points, St. Jude had 15 I think, Pisay had 10.7 or something. No one got any wave 1 question. Of wave 2, only Pisay got a question, and it was the easy question. The questions were hard, so this was only expected.

Near the end of wave 2, we actually had around seven choices left for the last person removed. So we just submitted all of them, and they were all wrong. Turns out the guy we just crossed off, sixteenth, was the correct answer. Oops.

Wave 3 came, and as soon as we saw the average question, we agreed it was doable. We set to work immediately, got the answer in within the first minute, which was great. Some other teams answered the average question as well. Pisay managed to answer the difficult very quickly. By the end of wave 3, Pisay was all the way up in second place.

Wave 4's easy was doable. It was taking a sum modulo thirty, so Vincent and I did that while Carabbay was working on average. We managed to submit answers to both questions pretty quickly, probably within the first two minutes. We were still in first place after wave 4 with about 43, but

Pisay was very close with 39 or so. I was beginning to hope we could win first place.

In wave 5, Pisay got the average relatively quickly. We got it quickly as well, once we realized we were missing the 1+ in the denominator; the other teams got it too. Only St. Jude got the easy question, and when asked later, they said it was a guess. You know, 2^5 is 32 and $3 + 2$ is 5 so $\frac{5}{32}$.

In the end, we had 46 points, followed by Pisay's 43. The scores were all close: GCC had 41 and St. Jude had 39.

	A	B
3	TEAM	TOTAL SCORE
4	Philippines Science High School - MC	43.8
5	Grace Christian College	41.4
6	Valenzuela City School of Mathematics	46.3
7	Saint Jude Catholic School	39.2

Figure 8: Final scores.



Figure 9: A picture of us!

8 EVENING

That meant we won. Carabbay and Vincent were ecstatic; I'll confess being excited too. I had to control Vincent's behavior, as he was getting a bit rousy about everything. Earlier, he played the МТАР hymn on the microphone!

Shaq and Kyle and I were at the judges' table, and Shaq asks if sir (Manuel) Loquias would teach in MOSC. He says he's busy, Shaq and Kyle tell me about him. We ask about the solution to the last series problem, and one of the judges showed us; it involved some known series with π s.

There was some more talking, of course. Steven and Balete-sempai were still watching. There was awarding, picture-taking. The roving trophy is awesome, by the way. Wish we would get to keep it.

Then we had to struggle to go home, because ma'am Gomas's phone is dead, and no one had any load to text kuya JR, so we had to go in and ask borrow one of the UPMMC's phones, thanks!

Then we went home, and talked about lots of things, and the traffic was bad because it was evening. We had drive-through and Carabbay demonstrated his skill at eating with only one hand free. We spoke about school, about Mathira, the future, how it all felt like a dream, because it was just so awesome to win, so nice.

Then we were home.

9 CONCLUSIONS

Well, we won, which was awesome. Ending my high school math contest life at a high note felt great. I have to admit it felt as good as socializing, which is the typical reason I went to math contests; winning here just felt good in itself, a sort of raw satisfaction that it all paid off, and that we got extremely lucky in several ways.

Let's admit it; luck played a big role in our win. We were lucky enough to have guessed and checked our way through lots of things, to exploit the rules system by choosing to go "solve fast" over "solve hard". The finals questions, we were lucky enough to solve quickly, stuff like that.

Although I originally started writing this report the day after Mathira, I ended up finishing it two weeks afterward, if only because I'm busy. There's a post-mortem in the appendix giving details of what happened afterward.

The thanks, and since it's final this time, I guess I should make it long:

- UP MMC for conducting the event a bit more smoothly than last year. For writing the problems, and showing us some good problems this year. I am of the belief good problems should be appreciated because

they're hard to write: Eliminations/6, Tier 3/3, Tier 3/C2, Tier 12/1, Wave 3/3.

- To the people who got me hooked on math in the first place: the vcsHS, now vcsMS, math department, and everyone who ever was a part. I often say I wouldn't be anywhere in math without you guys, and I really mean it. I would not grow mathematically if not for your love and support!

Off the top of my head, in chronological order, sir Acana for all the support; ma'am Soriano for being surprised I didn't know difference of two squares; ma'am Olivar (then ma'am Edillo) for trusting me enough to handle a class; sir Logronio for accommodating me in the math lab every day and feeding me books; ma'am Biñas and sir Agustin for being advisers; ma'am Balmes for taking lots of pictures. The new generation of math teachers: sir Frondoza, ma'am Imperial, sir Miclat, ma'am Gomas, sir Jayvee, sir Josh, for all kinds of support too. I hope I didn't forget anyone!

- The people who came before me in vcsHS were a big influence. Kuya Russelle is stuff of legend; his batch graduated right before our batch started. Thanks to him for being an example to follow, to being one of the first big names, for talking to me about math.

Ate Judy Ann. Ate Vine for humoring me mathematically. Kuya Gelo for spending time after class discussing math to me, thanks to your enthusiasm. Kuya Joshua. To the batch right before me, of course: ate Jade, kuya Reggie, kuya John Dave, ate Arz, even kuya Jezlor, kuya Bret, kuya Ian, ate Irish, ate Jazz, thank you for always being with us in training, teaching, and humoring us. I'm missing way more people, and it's my fault for not remembering.

- My contemporaries in vcsMS. In our batch, Axi, Louie, Nikki, Kelly, Leal, plus lots more: Aldrich, Harold, Kenneth, Paris, more. I'm happy to be part of the direct lineage of five batches of students after mine. In the order I remember: Jireh, Ryan, Arzel, Bas, Carabbay, Allen, Jasmin, Armand, Darelle, Ethel, Yuan, Marion, Drew, Ayrone, Matthew, Jeynor, Vincent, Jasper, Ralph, Aubrey, Evitha, Adrian, Bryan, Jerson, Zandy, Eugene. I definitely forgot a lot of people.
- To my peers. I'm not going to mention anyone, because I'm going to miss a lot of people, and I've mentioned your names countless times. Since I got into PMO nationals back in Grade 10, you welcomed me into your peer group despite not having come from MTC, where you guys all knew each other. And now we're all great friends!

Okay, maybe I can mention groups of people. To those in my batch, the Colleegers, and others. To those who came after. To those far away. And I'm referring to those who came before as well, and I suppose it won't hurt to mention: Balete–sempai, ate Hazel, sir Gari, sir Paco. Even further back, sir Carlo too.

- To those who came before, much before. This is way too indirect for Mathira, but I'll mention anyway since you did help me grow mathematically. Sir Eden and sir Louie and sir Chan Shio and sir Garces. It's going to be too much to mention everyone (plus my memory won't handle it), so maybe an overall thank you should suffice.
- Miscellaneous thanks that don't really fit in the other categories: ma'am Tafalla, ma'am Salonga, sir Tan, the Valenzuela City Government, MSP, MTG (sort of?). To the most important person I met from math contests, you know who you are. I really really hope I didn't miss anyone, but if you're reading this then you're already contributing by reading!

This year was a good year. I'm happy to end it on a high note.

A MECHANICS ANALYSIS

For future vcsms students. We're beginning with the (very high expectations) assumption that you made it to the semi-finals, and that your team makeup is similar to 2016–2018 teams.

- Overall remarks: the top four teams will typically end with scores in the five to seven hundreds, most of which are acquired from earlier tiers. The differences between fourth and next teams is typically only in the dozens. Given the question values are much higher than this in later tiers, it is necessary and sufficient to get a question that any of the top four do not get.
- General strategy: Speed is very important. No time to try more than one approach: stick to the first approach, split the work, go with it until you have a final answer, and check. Don't attempt to verify with a second approach. Strong emphasis on trial-and-error: abuse the fact answers to equations are almost always integral, try small cases, make wild conjectures.
- Early game, tiers 1–3: Good approximation for score is $12x$, where x is the question number. Believe it or not, these tiers are as important

as later tiers. Focus is especially important here: you do not want to miss a question that other teams get, as it could be costly in later tiers.

- Mid game, tiers 4–7: Approximation for score is $8x + 30$. Not much of interest will happen here, try to solve a problem or two that few teams will get.
- Late game, tiers 8–12: Approximation is $3x + 140$, which you can remember with $\pi \approx 3.14$. Given that the teams are usually closely bunched together, and that the problems are very difficult, not much will change here.

If you're in the top four: this part is relatively unimportant, as long as no team lower than you gets a correct answer. To prevent this, block by getting the answer whenever another team does, or score well in earlier rounds in order to not get pushed outside of top four. If you're currently top two or three, you can probably handle a team or two getting correct.

If you're not in the top four: given that the problems here are worth a lot, getting a problem that few teams higher than you get is typically enough to get you into the top four. You want to maximize probability of getting one problem correct, rather than maximize expected score.

- As an aside, the score approximations here are correct within ± 5 points. The actual curve is quadratic, and splitting it into three linear regions is actually really helpful mentally.

If you're lucky enough to make the finals:

- Overall remarks: the champion usually has scores in the forties or fifties, with differences between ranks around three to five points. It is probably not worth it to score less than one point on a question for this reason.
- Semi-finals scores are actually very important, and about half of the variance in scores in the finals will come from the semi-finals. So try to maximize semi-finals scores.
- You do *not* want to spam responses when the time is ending, rather, you should spam it near the 4 or 5 minute mark. The scoring functions are zero points when submitted with zero seconds left, and so are worth almost nothing when submitted near the time end. For this reason, only the difficult question is worth any decent number of points after 5 minutes.

- Speed is *very* important. Submitting a question, any question, within one minute is worth ten points. The difficulty doesn't matter for the purposes of this, so being fast is much more important than solving hard problems. For this reason, you also might want to ignore difficult questions.
- Easy questions decay very quickly: worth 5 points after 2.5 minutes, and 2.5 points after 3.5 minutes. After the 4 minute mark, easy questions are worth about 1 point.
- Average questions decrease linearly: one point every thirty seconds. The 3-point threshold is 4.5 minutes.
- Difficult questions decays very slowly, and then very quickly, so if you're lucky enough to solve them before the 5 minute mark, it's worth it. Solving it within the first 4.5 minutes ensures at least 5 points. After that, it decays really quickly.
- Again, this means speed is *very* important. Solving any question within the first minute is strictly better than solving any other question after the one minute mark, no matter the difficulty.

B POST-MORTEM

- Eliminations/6: Consider the odd terms and even terms separately and use Cassini's to get a telescope. I tried this, but it didn't seem to give the value computed empirically, so I'm not expounding on my solution.
- Eliminations/13: Ankan found a good solution for this. Let S be the sum. Then

$$\begin{aligned}
 S &= \cot\left(\frac{\pi}{7}\right) + \cot\left(\frac{2\pi}{7}\right) - \cot\left(\frac{3\pi}{7}\right) \\
 &= \cot\left(\frac{8\pi}{7}\right) + \cot\left(\frac{2\pi}{7}\right) + \cot\left(\frac{4\pi}{7}\right) \\
 &= \frac{\cos \frac{2\pi}{7}}{\sin \frac{2\pi}{7}} + \frac{\cos \frac{4\pi}{7}}{\sin \frac{4\pi}{7}} + \frac{\cos \frac{8\pi}{7}}{\sin \frac{8\pi}{7}} \\
 &= \frac{\cos \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7} + \sin \frac{2\pi}{7} \cos \frac{4\pi}{7} \sin \frac{8\pi}{7} + \sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \cos \frac{8\pi}{7}}{\sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7}}
 \end{aligned}$$

Observing the similarity with the triple angle sum formula for \cos , we then do

$$\begin{aligned} & \cot \frac{2\pi}{7} \cot \frac{4\pi}{7} \cot \frac{8\pi}{7} - S \\ &= \frac{\cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \cos \frac{8\pi}{7} - \cos \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7} - \sin \frac{2\pi}{7} \cos \frac{4\pi}{7} \sin \frac{8\pi}{7} - \sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \cos \frac{8\pi}{7}}{\sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7}} \\ &= \frac{\cos \left(\frac{2\pi}{7} + \frac{4\pi}{7} + \frac{8\pi}{7} \right)}{\sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7}} \\ &= \frac{1}{\sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7}} \end{aligned}$$

And we can find $\cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \cos \frac{8\pi}{7}$ and $\sin \frac{2\pi}{7} \sin \frac{4\pi}{7} \sin \frac{8\pi}{7}$ by known methods, Morrie's Law.

- Eliminations/15: The identity $\pi \cot \pi x = \sum_{n=-\infty}^{\infty} \frac{1}{x+n}$ kills this. To make it explicit, the sum is

$$\begin{aligned} S &= \sum_{k=1}^{\infty} \left(\frac{1}{24k-1} - \frac{1}{24k+1} + \frac{1}{24k-11} - \frac{1}{24k+11} \right) \\ 24S &= \sum_{k=1}^{\infty} \left(\frac{1}{k-\frac{1}{24}} + \frac{1}{-k-\frac{1}{24}} + \frac{1}{k-\frac{11}{24}} + \frac{1}{-k-\frac{11}{24}} \right) \\ 24S &= -\frac{1}{-\frac{1}{24}} - \frac{1}{-\frac{11}{24}} + \sum_{k=-\infty}^{\infty} \frac{1}{-\frac{1}{24}+k} + \sum_{k=-\infty}^{\infty} \frac{1}{-\frac{11}{24}+k} \\ 24S &= 24 + \frac{24}{11} + \pi \cot \left(-\frac{\pi}{24} \right) + \pi \cot \left(-\frac{11\pi}{24} \right), \end{aligned}$$

which can be evaluated with known methods.

- Tier 5/1: Ryan shares to me that they just guessed and checked this problem. The other two have a product of 28; Vieta's says the sum is 4. 9 and -2 don't work, but -9 and 2 work, because $-9 + 2 + 4 + 7 = 4$. Really rapidly expand $(x+9)(x-2)(x-4)(x-7)$.
- Tier 6/3: I finally memo'd the general formula for this after the contest. A square of side s inscribed in a triangle with side lying on base b with height h satisfies $\frac{1}{s} = \frac{1}{b} + \frac{1}{h}$.
- Tier 7/3: Apparently the intended solution is trial-and-error.
- Tier 8/1: The question's a bit vague, but apparently you're trying to count the number of positions of M, S, A in order. Which is balls-and-urns.

- Tier 9/2: Apparently $B(n, U(0, 1)) \sim U\{0, n\}$. The official solution intended to prove this with the beta integral. Here's a decomposition proof:

First, if $U \sim U(0, 1)$, then $1_{U < u} = B(1, u)$ by geometric arguments. Then if $U_i \sim U(0, 1)$, then $\sum 1_{U_i < u} = B(n, u)$. Now $B(n, U \sim U(0, 1)) = \sum 1_{U_i < U}$. Then the probability of a value of k is when U is the $k + 1$ th largest among the U_i . But this is uniform by symmetry.

In English: fix the probability p , treat each of the coins as a random number between 0 and 1. We count the coin if its value is less than p . The number of coins is thus the number of values less than p . The n coins/random numbers along with p form a set of $n + 1$ values between 0 and 1, and they're all uniformly random, so p is equally likely to be any rank in the list of the $n + 1$ values.

- Tier 10/1: Amusingly, the official solution did not use linearity. We were given out sheets of scratch paper with the solutions on the back, so yeah.
- Tier 11/3: This is a geometric random variable $X \sim G(p)$, and $E[X] = \frac{q}{p}$, which I should have memorized by now.
- Wave 1/3: This would actually be an interesting problem, if we were not rushing to solve it. Inverse tangent sum.
- Wave 3/3: Due to Kyle: The gap between the squares k^2 and $(k + 1)^2$ is $2k + 1$; so up to $k = 1010$ all of the numbers are covered. The last number covered is $\frac{1010^2}{2020} = 505$, after that, the numbers $1011^2 \dots 2020^2$ all cover distinct numbers. There are $2020 - 1011 + 1 = 1010$ of them, so the answer is $1010 + 506 = 1516$.