1 Find all pairs of natural numbers (n, k) for which $(n+1)^k - 1 = n!$. $[(1,1),(2,1),(4,2)]$

2 In the MMC *tambayan*, there is a square table of size 69×69. Mhay and Ann color each unit square either maroon or green, respectively, in such a way that each maroon square not lying on the edge of the table has exactly five green squares among its eight neighbors and each green square not lying on the edge of the table has exactly four maroon squares among its eight neighbors. Determine the number of maroon squares on the table. $[2116]$

3 Migel positions a ten-meter long ladder leaning upright against a wall, touching the edges of a box. The box itself is put up against the wall and measures two cubic meters. Determine the maximum height of the top of the ladder from the ground.

$$\left[\frac{1}{2}\left(\sqrt{100+\sqrt[3]{4}}+\sqrt[3]{2}+\sqrt{\left(\sqrt{100+\sqrt[3]{4}}+\sqrt[3]{2}\right)\left(\sqrt{100+\sqrt[3]{4}}-3\sqrt[3]{2}\right)}\right)\text{ m}\right]$$

4 Determine the sum of all monic quadratic polynomials of the form $x^2 + ax + b$ with integer roots, where 1, a, and b form an arithmetic progression. $[2x^2 + 8x + 14]$

5 $\triangle TON$ has 20, 28, and 35 as the lengths of its altitudes. Find the length of its largest side. $\left[\dfrac{245\sqrt{6}}{12}\right]$

6 The Fibonacci numbers are defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$. Evaluate the sum $\sum_{n=1}^{\infty} \frac{1}{F_n^2 + 1}$. $\left[\dfrac{5\sqrt{5}-3}{6}\right]$

7 Determine the number of solutions to the equation $x^2 - [x^2] = (x - [x])^2$ for $1 \le x \le 30$ where $[x]$ denotes the greatest integer less than or equal to x. $[871]$

8 Find all solutions to the following system of equations: $3(x-2)^2 - 4y - z = -3$, $11x - \frac{8y}{3} - 2z = 30$, and $3x - 2y - 6z = -12$. $\left[(4,3,3), \left(\dfrac{113}{18}, \dfrac{259}{18}, \dfrac{37}{108}\right)\right]$

9 Let A be the set of positive integers less than or equal to 2018 and B be the set of integers not divisible by integers greater than 2 but less than 10. Find the sum of the elements in $A \cap B^{\mathsf{C}}$. $[698\,337]$

10 Mong has 101 marbles numbered from 1 to 101. The marbles are divided over two baskets A and B. The marble numbered 40 is in basket A. This marble is removed from basket A and put in basket B. The average of all the numbers on the marbles in A increases by $\frac{1}{4}$. The average of all the numbers of the marbles in B increases by $\frac{1}{4}$ too. Determine the number of marbles originally in basket A. $[73]$

11 Spheres of unit radius are being stacked in the form of a square pyramid. At the bottom, there is a layer in the form of a square with 2018×2018 spheres. On top of that layer comes the next layer with 2017×2017 spheres and so on. Determine the height of the pyramid. $[2017\sqrt{2} + 2]$

12 Find the maximum value of $|x - y|$ if $x(7x-1) + y(7y-1) = 0$.* $\left[\dfrac{1}{7}\right]$

13 Find the exact value of $\cot\frac{\pi}{7} + \cot\frac{2\pi}{7} - \cot\frac{3\pi}{7}$. $[\sqrt{7}]$

14 Determine all triplets (x, y, z) of natural numbers with z as small as possible for which there exist natural numbers a, b, c, and d satisfying $x^y = a^b = c^d$ with $x > a > c$, $z = ab = cd$, and $x + y = a + b$. $[(8,8,48)]$

* Question voided.

15 Evaluate the sum $\sum_{k=1}^{\infty}\left(\frac{1}{24k+11}+\frac{1}{24k+1}-\frac{1}{24k-11}-\frac{1}{24k-1}\right)$.

$$\left[\frac{\pi\sqrt{2+\sqrt{3}}}{6}-\frac{12}{11}\right]$$