MMC 2015

Sectoral Level, Category B

If you have any corrections or additions, please contact me at `cj@cjquines.com`, or through my Facebook account, Carl Joshua Quines.

Grade 7.

E1. What number must be added to -5×-10 to produce -7×8? [-106]

E2. Find the area of a triangle with base $2x+2$ cm and height $2x+1$ cm. [$2x^2+3x+1$ sq. cm]

E3. The sum of two primes is 16. What is the largest possible value of their product? [55]

E4. In a quadrilateral, 3 interior angles have measures $87°$, $89°$, and $91°$. Find the fourth angle. [$93°$]

E5. What is the sum of $(x+3)(x-3)$ and $(x+3)^2$? [$2x^2+6x$]

E6. By what percentage will the volume of a cylinder increase or decrease, if the radius is decreased by 10% and the height increased by 10%? [decrease by 10.9%]

E7. With what integer must -54 be divided by to get a remainder of 11 and a quotient of -5? [13]

E8. The sum of n numbers is $50n$. If each number is increased by 50, what is the new sum? [$100n$]

E9. How many integers n are there for which $|n-5| < 11$? [21]

E10. The sum of two numbers is 10 and the sum of their reciprocals is $\frac{5}{3}$. Find their product. [6]

E11. What is the largest integer smaller than $\sqrt{125}$? [11]

A1. If $x^2-1=4$, evaluate $\frac{x^6+3x^4}{x^4+2x^2+5}$. [5]

A2. The base of a rectangular box has dimensions 30 cm and 45 cm. A cube, with side length 6 cm, is submerged in the water inside. By how much will the water level rise? [$\frac{4}{25}=0.16$ cm]

A3. A rectangle has area 48 sq. units. The midpoints of two adjacent sids, and the common vertex of the other two sides, are connected to form a triangle. Find its area. [18 sq. units]

A4. The angles of a triangle are $(4x-13)°$, $(4x+21)°$, $(5x-23)°$. Find the largest angle. [$81°$]

A5. A right triangle has legs 5 cm and 12 cm long. A bug, as it travels around it, is always 3 cm away from the triangle. What is the length of the bug's path? [$6\pi+30$ cm]

A6. The product of two numbers is $98,000x^5y^4$ and their greatest common factor is $70x^2y$. What is their least common multiple? [$1400x^3y^3$]

D1. A car and a truck, 500 km apart, drive towards each other until they meet, with the car driving at 90 km/hr, and the truck at 60 km/hr. How fast should the car drive for its return trip to take 2 hours? [150 km/hr]

D2. If n is a constant such that $||x-2|-3|=n$ has exactly 3 distinct roots, what are these roots? [-4, 2, 8]

D3. If $p+q \neq 0$, solve for x: $\frac{x-3p}{q} + \frac{x-2q}{p} = 5$. [$x = 3p + 2q$]

D4. The sum of 4 numbers is 75. If the number 4 is added to the first, subtracted from the second, multiplied to the third, and used as the divisor for the fourth, the resulting numbers are al equal. What are the 4 original numbers, in order? [8, 16, 3, 48]

D5. The figure shown is formed by joining 4 semicircles. If $AC = 18$ and $BD = 14$, find the area of the enclosed region. [63π sq. units]

D6. Arrange these numbers in ascending order: $p = 3^{120}, q = 4^{96}, r = 5^{72}, s = 6^{48}$. [$s, r, p, q$]

C1. What number is exactly midway between -35 and 3 on the number line? [-16]

C2. What must be multiplied to $x+3$ so the product is 10 less than the product of $x+1$ and $3x+4$? [$3x-2$]

C3. The medians to two legs of a right triangle are 6 and 7 units long. How long is the hypotenuse? [$2\sqrt{17}$ or $\sqrt{68}$]

DoD. Compute: $\frac{1+2}{1+2+3} \times \frac{1+2+3+4}{1+2+3+4+5} \times \cdots \times \frac{1+2+\cdots+14}{1+2+\cdots+15}$. [$\frac{1}{8}$]

Grade 8

E1. What is the slope of the line with equation $4x + 3y - 2 = 0$? [$-\frac{4}{3}$]

E2. Arnold is 7 years older than her sister Rica. If Arnold will turn 36 seven years from now, how old is Rica now? [22 years old]

E3. Solve for x in the equation $2(x+2) + x = 2x + 3$. [-1]

E4. Find the area of the traingle bounded by the line $2x + y = 4$ and the coordinate axes. [4 sq. units]

E5. Factor completely: $x^3y - xy^3$. [$xy(x-y)(x+y)$]

E6. If the base angles of an isosceles triangle measure $2x$ and $3x - 20$ degrees, find the measure of the vertex angle. [$100°$]

E7. If $f(x) = x^2 - 3x + 1$, what is $f(-1) + f(0)$? [6]

E8. If the sides of a triangle have integral lengths, and its perimeter is 13 cm, what is the largest possible length of one side? [6 cm]

E9. What is the probability of getting a sum of 5 in rolling a pair of dice? [$\frac{1}{9}$]

E10. If $xy = 13$ and $x + y = 10$, what is $x^2 + y^2$? [74]

E11. Find the distance between the points (2,1) and $(-3, 4)$. [$\sqrt{34}$ units]

A1. Find the measure of the acute angle between the hour and the minute hands at 12:15 PM. [$82.5°$]

A2. If $\frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{(x+1)^3} = \frac{A}{(x+1)^3}$, what is A? [$x^2 + 3x + 3$]

A3. Christine can paint a whole table in 50 minutes, while Vicky can do it twice as fast as Christine. If they continuously work together, how long will it take them to finish 15 such tables? [250 min]

A4. If $ab = 3$ and $a - b = 7$, what is $a^4b^2 - 2a^3b^3 + a^2b^4 + a^2b - ab^2$? [462]

A5. One leg of an isosceles right triangle measures 12 cm. What is the area of the circle that passes through the vertices of the triangle? [72π sq. cm]

A6. Which integer values of x satisfy $3x + 1 < 2x + 11 \leq 4x - 3$? [7, 8, 9]

D1. If n is the largest negative integer that satisfies $17 < |2x - 1| < 20$, what is $2 - 3n$? [29]

D2. What is the equation (in the form $y = mx + b$) of the perpendicular bisector of the segment joining the points $(4, -2)$ and $(-1, 4)$? [$y = \frac{5}{6}x - \frac{1}{4}$]

D3. James jogs every morning, while Dina cycles on the same route. If Dina's speed is 3.5 times that of James, and Dina starts 2 hours after James, how many minutes does Dina cycle before she overtakes James? [48]

D4. With what polynomial must $6x^4 - 2x^3 + x^2 + x - 5$ be divided to get a quotient of $2x^2 + 5$ and a remainder of $6x + 30$? [$3x^2 - x - 7$]

D5. Let $ABCD$ be a square, and let E, F, G and H be the midpoints of sides CD, AD, AB and BC, respectively. The segments AE, BF, CG and DH create a smaller square inside $ABCD$. If the area of this smaller square is 1.5 sq. units, what is the area of ABCD? [7.5 sq. units]

D6. The sides of a right triangle are x, $x + y$, $x + 2y$, where x and y are positive numbers. What is the ratio of y to x? [$1 : 3$]

C1. What is the equation of the line (in the form $y = mx + b$) that passes through $(2, -3)$ and the origin? [$y = -\frac{3}{2}x$]

C2. Two consecutive interior angles of a parallelogram measure $7x + 48$ and $2x + 90$ degrees. What is $6x$? [28]

C3. A chemist has two alcohol solutions of the same kind but of different strengths, one with 35% alcohol and another with 50% alcohol. How many liters of each solution must be mixed to produce 60 liters of solution with 40% alochol? [40 L of 35% alcohol, 20 L of 50% alcohol]

DoD. Find all positive integers a, b, c and d that satisfy the equation $a = b^2$, $c^3 = d^2$, and $c - a = 49$. [$a = 24^2, b = 24, c = 25^2, d = 25^3$]

Grade 9.

E1. If $\frac{x}{y} = 2$, find $\frac{x^2-y^2}{x^2+y^2}$. [$\frac{3}{5}$]

E2. In a cube with sides of length 1 cm, one vertex is denoted by A. What is the sum of the distances from A to each of the other vertices of the cube? [$3 + 3\sqrt{2} + \sqrt{3}$ cm]

E3. Find two positive numbers in the ratio 7:12 so that the bigger number exceeds the smaller by 10. [14, 24]

E4. Benigno buys four new tires and a new spare tire for his car. He rotates his tires so that after driving 10 000 km, every tire has been used for the same number of kilometers. For how many kilometers was each tire used? [8 000]

E5. The longest side of a triangle measures 12 cm, and the altitude to this side is 4 cm. What is the length of the shortest side if the altitude to this side measures 8 cm? [6 cm]

E6. A quadratic function $f(x)$ satisfies $f(0) = 30$ and $f(2) = 0$. Determine all the zeros of $f(x)$. [2 and 15]

E7. Find the sum of the reciprocals of two numbers, given that the sum of the two numbers is 18 and their product is 6. [3]

E8. On square ABCD, one side measures 1 cm, E is the midpoint of AB and F is the point of intersection of CE and the diagonal BD. How long is FB? [$\frac{\sqrt{2}}{3}$ cm]

E9. Determine the quadratic function $f(x)$ that satisfies $f(1) = f(2) = 0$ and $f(0) = 4$. [$f(x) = 2x^2 - 6x + 4$]

E10. If x and y are positive numbers such that $x^2 - 3y^2 = 2xy$, find $\frac{x}{y}$. [3]

E11. A triangle with sides in the ratio 3:4:5 is inscribed in a circle of radius 5 cm. Find the area of the triangle. [24 sq. cm]

A1. Find a number x that makes $\frac{x+7}{2(x+14)}$ equal to $\frac{5}{8}$. [-42]

A2. An MRT train 0.2 km long travels at a steady speed of 24 kph. It enters a tunnel 1 km long at exactly 3:00 PM. At what time will the tail-end of the train come out of the tunnel? [3:03 PM]

A3. A rectangle is formed by placing three congruent rectangles as shown in this diagram. What is the area of the larger rectangle, if the shorter side of the small rectangle measures 2 cm? [24 sq. cm]

A4. If $f(2x+1) = 4x^2 + 2x - 6$, what are the zeros of $f(x)$? [$x = -2, x = 3$]

A5. Three identical equilateral corners of an equilateral triangle with side of length 1 cm is to be cut off to reduce the area by one-half. how long should be the side of the cut-off corners? [$\frac{\sqrt{6}}{6}$ cm]

A6. A rectangle has a diagonal of length $2\sqrt{5}$ cm and area 8 sq. cm. What is the perimeter of the rectangle? [12 cm]

D1. Solve for a number x satisfying $\frac{2x^2-2x+4}{2x^2-2x+3} = \frac{x^2+2x}{x^2+2x-1}$. [$x = 2$]

D2. The sides of an isosceles triangle measure $5x+3$, $3x+7$ and $2x+15$, where x is some number. What is the largest possible perimeter of the triangle? [105]

D3. For what value(s) of a will the quadratic equations $x^2 - ax + 2 = 0$ and $x^2 - 2x + a = 0$ have a common real solution? [$a = -3$]

D4. On his way from a vacation, a man figured that he will be home by 11:00 PM if he drives at 60 kph. If he drives at 40 kph he will be home by 1:00 AM. How fast must he drive if he wants to be home by 12:00 midnight? [48 kph]

D5. The area of rectangle ABCD is 24 sq. cm and E is the midpoint of CD, F is the point of intersection of the diagonal AC and segment BE. Find the area of triangle EFC. [2 sq. cm]

D6. The diagonal of a regular pentagon divides an interior angle into 2 angles. Find the measure of the smaller angle. [$36°$]

C1. The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^2+bx+c$ are all equal. What is their common value? [a]

C2. Determine all values of x and y which make xy, $\frac{x}{y}$ and $x - y$ equal to each other. [$x = -\frac{1}{2}, y = -1$]

C3. A point O lies inside equilateral triangle ABC so that $AO^2 + BO^2 = CO^2$. Find the measure of $\angle AOB$. [$150°$]

DoD. How many non-congruent right triangles have sides of integer lengths and have areas numerically equal to three times their perimeters? [6]

Fourth Year.

E1. What is the next term in the arithmetic sequence whose first two terms are -18 and then 19? [56]

E2. Express $-3 + \log_2 x + 3\log_2 y$ as a single logarithm with base 2. [$\log_2(\frac{xy^3}{8})$]

E3. What should be the value of c if the point where $f(x) = -x^2 + 2x + c$ attains its maximum, is also a zero of f? [-1]

E4. What is the reciprocal of $\sqrt[3]{9} - 2$ in simplest terms? [$3\sqrt[3]{3} + 2\sqrt[3]{9} + 4$]

E5. If $\log_x 25 = 4$, what is $\log_2 5x$? [$\frac{1}{4}$]

E6. What is the smallest positive integer n which satisfies $n^20 \geq 5^30$? [12]

E7. Find k such that the circle $x^2 + y^2 = 4$ intersects the parabola $y = x^2 + k$ in exactly three points. [-4]

E8. Find the value of θ in $[0°, 180°]$ assuming $\frac{\sin 20°}{\cos 70°} = \tan\theta$. [45°]

E9. The cube of a number x equals 2015^{15}. What is the product of x and the square of 2015^{14}? [2015^{33}]

E10. Find the smallest positive value of θ such that $\sin(x - \theta) = \cos\theta$. [$\frac{3\pi}{2}$]

E11. Which real number is not attained by the function $f(x) = \frac{2x+4}{1-6x}$? [$-\frac{1}{3}$]

A1. Find the domain of $f(x) = \frac{2-\log_3 x}{3-\log_2 x}$. [$(0, 8) \cup (8, \inf)$]

A2. Find the 12th term of the harmonic sequence $\frac{3}{2}, \frac{3}{7}, 4, \ldots$ [$\frac{1}{19}$]

A3. What is the numerical value of $\log_8(\cos(\frac{7pi}{4}))$? [$-\frac{1}{6}$]

A4. An isosceles right triangle, with area n sq. cm, has a hypotenuse which also measures n cm. Find n. [4]

A5. A linear function f is such that $f(2015) - f(2005) = 100$. What is $f(2051) - f(2015)$? [360]

A6. If $f(1 + \ln x) = 2x$, what is $f(x)$? [$2e^{x-1}$ or equiv.]

D1. The numerical value of $\log_3 2015$ is in between which consecutive integers? [6 and 7]

D2. Evaluate the sum $\sin 30° + \sin 60° + \sin 90° + \cdots + \sin 510° + \sin 540°$. [$2 + \sqrt{3}$]

D3. The roots of $x^3 + ax^2 + bx - 18$ are prime numbers. Find $a + b$. [13]

D4. Solve for x in the equation $\sqrt{2x+3} + \sqrt{-x} = 2$. [$-1, -\frac{1}{9}$]

D5. How many positive integers n are there such that $2n$ and $\frac{n}{3}$ are three-digit integers? [67]

D6. For $n \geq 2$, the nth term of the sequence equals the sum of all the terms before it. If the eighth term is 320, what is the first term of the sequence? [5]

C1. Solve for x: $(\ln x)^2 = 3 + \ln x^2$. [$e^3, \frac{1}{e}$]

C2. For which real x can we sum the geometric series $x, -1, \frac{1}{x}, \ldots$ to infinity? [$x > 1$ or $x < -1$]

C3. If p, q, r are the prime factors of 2015, find $\log_7(p + q + r)$. [2]

DoD. Find all possible values of x so that the sequence $x + 6, 6, x + 1$ becomes geometric. [-10, 3]