

# MMC 2016

Regional Level, Category B

February 19, 2016, March 4, 2016

Disclaimer: inexact wording, may not be 100% accurate. I can vouch wording for grade 6 is pretty accurate. If you have any corrections or additions, please contact me at [cj@cjquines.com](mailto:cj@cjquines.com), or through my Facebook account, Carl Joshua Quines. It is hoped that an official copy will be released in the near future, but MMC did not release a copy of the regional questions last year, so . . . I will be relying on the transcription of my teacher for grades 7 and 8, as I was unable to attend the actual competition. I do not have a copy of the grade 9 questions.

## Grade 6.

- E1. The sum of three consecutive whole numbers is 99. The largest number in the sum is 34. What is the smallest number is this sum? [32]
- E2. Twenty poles are placed in a straight line. The distance between any two consecutive poles is 3 m. What is the distance between the first and the tenth pole? [27 m]
- E3. Two circles have a common center. The radius of the larger circle is 7 cm and the radius of the smaller circle is 3 cm. What is the area between them? Give your answer in terms of  $\pi$ . [ $40\pi$  sq. cm]
- E4. In the number 500500 how many times greater is the value of the leftmost 5 than the value of the rightmost 5? [1000]
- E5. What is the value of  $(1 \times 2016) + (2 \times 2016) + (3 \times 2016) + (4 \times 2016)$ ? [20160]
- E6. When a certain number is divided by 8, the quotient is 4 and the remainder is 2. What is the number? [34]
- E7. How many prime numbers are factors of 30? [3]
- E8. On a sheet of paper, a line is drawn through the center of a square drawn on the paper. How many times does this line intersect the sides of the square? [2]
- E9. In what month does the one hundredth day of the year occur? [April]
- E10. What number should be on the blank?  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 2 \times 12 \times \dots$  [30]
- A1. How many of the integers from 2 to 999 are squares of whole numbers? [30]
- A2. My dog was 120 m from home and my cat was 70 m from home. I called them and they both ran directly toward me. If my dog ran twice as fast as my cat, how far from home was my cat when my dog reached home? [10 m]
- A3. What is the fewest number of squares each with a perimeter of 4 cm that will completely cover a square with a side of 4 cm? [16]
- A4. If 10% of 10% of a certain number is 2, what is the number? [200]
- A5. Rick multiplied his age by 2, added 20, divided by 2, and then subtracted his age. What number did Rick finally get? [10]

- D1. A circle and a square intersect so that two sides of the square are radii of the circle. If a side of the square is 3 cm, what is the area of the region inside the square but outside the circle?  $[9 - \frac{9\pi}{4} \text{ sq. cm}]$
- D2. Lloyd was born  $6\frac{1}{4}$  years after Alex. The sum of their ages is  $66\frac{11}{12}$  years. How old is Alex?  $[36\frac{7}{12} \text{ years}]$
- D3. On a shelf,  $\frac{3}{8}$  of the books are geometry,  $\frac{1}{3}$  of the remaining books are algebra, and the rest are statistics. If there are ten statistics books, find the total number of books on the shelf.  $[24]$
- D4. A ferris wheel can accommodate 60 people in 20 minutes. How many people can ride the ferris wheel in 4 hours?  $[720 \text{ people}]$
- D5. If the month of July has 5 Tuesdays, and three of them fall on even-numbered days, what is the date of the third Tuesday in July?  $[\text{July } 16]$
- C1. A whole number is greater than 20 and less than 2000. What is the smallest possible value of the sum of all its digits?  $[1]$
- C2. In a list of 200 numbers, each number after the first number is 4 more than the number that comes before it. What is the difference between the first and the last number of this list?  $[796]$
- C3. The base of a triangular prism is a right triangle with legs 3 cm and 4 cm, respectively. If the height of the prism is 5 cm, what is its total surface area?  $[72 \text{ sq. cm}]$
- DoD. Rica owns 3 houses and makes money by renting them out. She charges 3 times as much per month for the second house than the first. The monthly rent for the third house is 18000 pesos less than the sum of the monthly rents for the first two houses. The first house was vacant for five months, but otherwise rents were received every month from the tenants of the three houses. Rica had a total rent receipt of 603,000 pesos for the year. How much per month was the rent of the third house?  $[18000 \text{ pesos}]$

### Grade 7.

- E1. What is the result when  $5 \times -6$  is subtracted from  $-9 \times -2$ ?  $[48]$
- E2. What is the value of  $\frac{\frac{3}{4} + \frac{1}{6}}{4 - \frac{1}{3}}$ ?  $[\frac{1}{4}]$
- E3. The number of students increased by 10% this year. If there are 1540 students this year, how many students were there last year?  $[1400]$
- E4. If  $x$  is twice as far from  $-2$  as it is from 1, what is  $x$ ?  $[0, 4]$
- E5. If  $\sqrt{x^2} = 5$ , what is  $x$ ?  $[5]$
- E6. Expand and simplify:  $(2x + z - 1)(2x - z + 1)$ .  $[4x^2 - z^2 + 2z - 1]$
- E7. Simplify  $\frac{95^2 - 25^2}{7}$ .  $[1200]$
- E8. Factor completely:  $x^2 - 4 + 3(x + 2)$ .  $[(x + 1)(x + 2)]$
- E9. What is the remainder when  $x^3 - 5x + 3$  is divided by  $x - 2$ ?  $[1]$
- E10. Ms. Salvador has 40 students. When she asked if they studied last weekend, 23 said they studied last Saturday and 28 said they studied last Sunday. If 15 students said they studied both days last weekend, how many students did not study on both Saturday and Sunday?  $[4]$
- A1. Two circles share a common center and have integer radius. If the area between the two circles is  $19\pi$  sq. cm, what is the perimeter of the smaller circle?  $[18\pi \text{ cm}]$
- A2. The sum of 2 numbers is 12 and the difference of their squares is 96. What is the smaller number?  $[2]$

- A3. If Gabi is  $x$  years old and is  $\frac{3}{7}$  as old as Free, in how many years will Gabi be half as old as Free? [ $\frac{x}{3}$  years]
- A4. Dante is taking a 150 km trip driving at 40 kph in the last 2 hours. What should be his speed for the remainder of the 3 hours to have an average of 50 kph? [70 kph]
- A5. A pitcher contains 440 mL of water mixed with 60 mL of juice concentrate. If 150 mL of the mixture is poured out and replaced with water, so that the volume is the same. What is the percentage of concentrate in the new mixture? [8.4%]
- D1. How many positive integers less than 216 are divisible by 28 but not by 12? [48]
- D2. The sum of two numbers is 9 and their product is 25. What is the sum of their cubes? [54]
- D3. If the sum of fifty consecutive integers that come after  $n$  is 6525, what is the sum of fifty consecutive integers before  $n$ ? [3975]
- D4. If the sum of the digits of a two digit number is subtracted from the number, the units of the digit number is 4. How many two digit numbers have this property? [10]
- D5. In the figure, each vertex of the square is joined to the midpoint of the opposite side. What fraction of the square is shaded? [ $\frac{1}{5}$ ]

### Grade 8.

- E1. There is 20 peso, 50 pesos, 100 pesos and 500 peso bills in a box. You select one bill at random. What is the probability that the amount remaining in the box is less than 650 pesos? [ $\frac{3}{4}$ ]
- E2. For which values of  $a$  is the point  $(a - 2, 1 - a)$  on the second quadrant? [ $a < 1$ ]
- E3. If the inverse of  $\frac{n}{3}$  is an integer, then  $\frac{n}{6}$  is an integer. True or false? [True]
- E4. If  $3x - 4y = 9$ , what is the value of  $8y - 6x$ ? [-18]
- E5. Transform the following statement to an if-then statement: all equilateral triangles are acute.  
[If a triangle is equilateral, then it is acute.]
- E6. Factor completely:  $y^4 - 81x^4$  [ $(y^2 + 9x^2)(y - 3x)(y + 3x)$ ]
- E7. If 3 is the x-intercept of the line  $x - 3y + 2c = 0$ , what is  $c$ ? [ $-\frac{3}{2}$ ]
- E8. Among the following points indicate all that are solutions of the inequality  $5x + 2y > 17$ :  $(3, 1), (3, 2), (4, -2), (5, -3)$ . [(3, 2), (5, -3)]
- E9. Find the value of  $\frac{78^2 - 96^2}{78 + 96}$ . [-18]
- E10. The probability of selecting a female from a group of 50 people is 0.6. How many males are in the group? [20]
- A1. Let  $ABCDE$  be a pentagon with parallel sides  $AB$  and  $BE$ . Angle  $ADC$  is  $146^\circ$  and angle  $EDC$  is  $120^\circ$ . Find angle  $BCD$ . [ $94^\circ$ ]
- A2. Give the slope-intercept form of the line whose y-intercept is twice the x-intercept and is passing through  $(2, -3)$ . [ $y = 2x + 1$ ]
- A3. At the given instant container  $A$  has 6 liters of water,  $B$  has 12 liters of water. If the volume of container  $A$  is increased by 2 liters per minute, and  $B$  is decreased 2.5 liters per minute, what will be the volume in both containers at the instant they contain an equal amount of water? [ $\frac{26}{3}$  liters]
- A4. Let  $S = \{1, 2, \dots, 1000\}$ . If a number is selected at random, what is the probability that it is divisible by 17? [ $\frac{29}{500}$ ]

- A5. In triangle  $ABC$ , angle  $A$  is  $34^\circ$ , and the exterior angle of vertex  $B$  is complementary to angle  $C$ . Find angle  $B$ .  
[118°]
- D1. Point  $D$  is inside triangle  $ABC$  such that  $BD$  and  $CD$  are angle bisectors of angle  $ABC$  and angle  $ACB$ , respectively.  $BD$  is equal to  $CD$ . Find angle  $BDC$  if angle  $BAC$  is  $32^\circ$ .  
[163°]
- D2. In the system  $8y - 3x \leq 16, 3x + 8y \geq -18$  for which solution  $(x, y)$  is  $x + y$  least?  
[(-6, -4)]
- D3. Triangle  $ABC$  has vertices  $A(-2, 6), B(4, 4), C(4, a)$ . Find the values of  $a$  such that the area of triangle  $ABC$  is equal to 21.  
[11, -3]
- D4. The y-intercept of two perpendicular lines are  $b$  and  $-2$ . The x-intercept are 3 and  $a$ , respectively. If  $a + b = 15$ , find the slope of both lines.  
[-3,  $\frac{1}{3}$ ]
- D5. You tossed three unbiased coins then remove all tails in the result, and you tossed the remaining coins. What is the probability that you tossed exactly three tails in the remaining coins?  
[ $\frac{27}{64}$ ]

### Grade 10.

- E1. The first term and common ratio of a geometric sequence is 2016 and  $\frac{1}{2}$  respectively. What is the fourth term?  
[252]
- E2. The area of the circle circumscribing a right triangle is  $45\pi$  sq. cm. Find the hypotenuse of the triangle.  
[ $6\sqrt{5}$  cm]
- E3. What is the vertex of the parabola  $y = (x + 4)(2 - x)$ ?  
[(-1, 9)]
- E4. Find the remainder when  $x^5 - x$  is divided by  $2x + 1$ .  
[ $\frac{15}{32}$ ]
- E5. In a circle, 2 internal chords divide each other into 2 segments. One segment of a chord is 3 cm and 8 cm. While the other segment of the second chord is 2 cm longer than the other. Find the length of the second chord.  
[10 cm]
- E6. If  $x$  and  $y$  are positive numbers such that  $x^2 + y^2 = 8$  and  $xy = 2$ , find  $x + y$ .  
[ $2\sqrt{3}$ ]
- E7. A circle passes through  $(0, 0), (6, 0)$  and  $(0, 5)$ . What is the center of the circle?  
[(3,  $\frac{5}{2}$ )]
- E8. What is the largest value of  $x$  that satisfies the inequality  $3x^2 + x^3 \leq 0$ ?  
[0]
- E9. The graph of the function  $f(x) = a(x + 1)^3$  passes through the point  $(1, 4)$ . What is  $f(3)$ ?  
[32]
- E10. Simplify  $(102 \times 98) - (105 \times 95)$ .  
[21]
- A1. The geometric mean and harmonic mean of two numbers are 6 and  $\frac{72}{13}$ . What is the sum of the two numbers?  
[13]
- A2. Two sides of a triangle are 8 cm and 10 cm, and the angle between them is  $60^\circ$ . Find the perimeter of the triangle.  
[ $18 + 2\sqrt{21}$  cm]
- A3. If one root of  $2x^3 + 3x^2 = 5x + k$  is  $-1$ , find the other roots.  
[ $\frac{3}{2}$  and  $-2$ ]
- A4. Two opposite vertices of a square lie at  $(0, 4)$  and  $(7, 3)$ . Find the other vertices.  
[(3, 0), (4, 7)]
- A5. Solve for  $x$  in the equation  $3 + \sqrt{x} = \sqrt{3x - 11}$   
[25]
- D1. A tank contains 150 L of mango puree and 50 L of water. Then 40 L of the mixture is removed and is replaced by 40 L of water. What is the percentage of mango puree in the final mixture?  
[60%]
- D2. Find the point on the x-axis equidistant to the points  $(8, 1)$  and  $(0, 5)$ .  
[( $\frac{5}{2}$ , 0)]

- D3. A third-degree polynomial satisfies  $P(0) = -3$  and  $P(1) = 4$ . When  $P(x)$  is divided by  $x^2 + x + 1$  the remainder is  $2x - 1$ . In the same division, what is the quotient? [3x - 2]
- D4. In how many ways can 3 distinct numbers be chosen from the set  $\{1, 2, 3, \dots, 10\}$  such that the sum of the numbers is odd? [60]
- D5. Find the sum of all 3-digit odd positive integers. [247500]