MMC 2016

Sectoral Level, Category B

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Disclaimer: inexact wording, may not be 100% accurate. I can vouch that the wording for grades 8 and 10 are sort of accurate, but not for grades 7 and 9. If you have any corrections or additions, please contact me at cj@cjquines.com, or through my Facebook account, Carl Joshua Quines. It is hoped that an official copy will be released in the near future.

Grade 7.

E1. What must be subtracted from -9×6 to obtain 70? $[-124]$

E2. What is the remainder when $2x^2 - 20$ is divided by $x + 3$? $[-2]$

E3. The length of a rectangle is 50% longer than its width. If the area of the rectangle is 54 sq. m, what is its perimeter? [30 m]

E4. A square is inscribed in a circle of radius 3. What is the area of the square? [18 sq. units]

E5. What is the sum of $(2x+3)(2x-3)$ and $3(2x+3)$? $[4x^2 + 6x]$

E6. A pentagon is formed by an equilateral triangle sharing a side with a square. What is the average of the measure of the smallest and biggest angles of the pentagon? $[105°]$

E7. The sum of the squares of two numbers is 76 and their product is 6. What is their difference? [8]

E8. A polygon has 14 diagonals. How many sides does the polygon have? [7]

E9. What is the value of $2^0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}$? $[\frac{31}{16}]$

E10. What is the value of $\sqrt{1.69 \times 10^{-16}}$? $[1.3 \times 10^{-4}]$

A1. If 201625AB is divisible by 12, what is the smallest two digit number AB? [20]

A2. In rectangle $ABCD$, M is the midpoint of diagonal BD. If $AB = 4$ and $\angle AMB = 120°$, what is the length of diagonal BD? [8]

A3. The average weight of the pupils in a class is 16 kg. If the average weight of the girls is 14 kg, and the average weight of the boys is 19 kg, what is the ratio of the number of boys to the number of girls? [2:3]

A4. Of the 40 students in a class, 28 are wearing watches while 16 are wearing eyeglasses. Find the greatest number of students who are not wearing a watch or eyeglasses. [12]

A5. Tom and Jerry are cycling towards each other at the same time. Tom's speed is five-sixth than that of Jerry. If they meet when Jerry has cycled 42 km, how far apart were they at the beginning? [77 km]

D1. A square picture is in a square frame. The side lengths of the frame and the picture are integers and the border is more than 1 inch. If the area of the border is 51 sq. in, what is the area of the picture? [49 sq. in]

D2. In how many ways can Daryl distribute 20 candies to his 3 friends if each friend must receive at least 5 candies? [21 ways]

D3. Three identical pails were filled with water. Pail A had a weight of 3.2 kg when it was two-thirds full. Pail B had a weight of 1.95 kg when it was one-fourth full. What fraction of pail C is filled if it has a weight of 3.6 kg? $[\frac{4}{5}]$

D4.

D5.

Grade 8.

E1. A triangle has many "centers". How do we call the center of the triangle that is determined by its three altitudes? [Orthocenter]

E2. Evaluate $20^3 - 16^3$. [3904] (3984 was said)

E3. Find all prime numbers that are more than 2016 but less than 2050. [2017, 2027, 2029, 2039]

E4. Find the sum of all positive divisors of 2016. [6552]

E5. What is the sum of the mean, the median and the mode of the numbers 2, 0, 1, 4, 2, 0, 1, 5, 2, 0, 1, 6, 2, 0, 1, 7? [6.625] (6.75 was said)

E6. A motorboat can travel at an average speed of 11 kph in still water. Travelling with the current, the motorboat can travel 77 km down the river in the same amount of time it takes to travel 44 km upstream against the current. Find the speed of the current. [3 kph]

E7. If $f(x) = 3x^2 + 4x - 7$, what is $f(f(-1))$? [153]

E8. Two standard dice are rolled. What is the probability of the sum on the two dice is greater than or equal to 8? $[\frac{5}{12}]$

E9. Find the area of the circle that is inscribed in a 60° sector of a circle with radius 12 units. [16π sq. units] (24π was said)

E10. Find the radius of the circle inscribed in a triangle of sides 20, 21 and 29 cm. [6 cm]

A1. In a deck of cards, what is the probability of choosing two red cards? $[\frac{25}{102}]$

A2. In triangle ABC, $AC = 5$, $BC = 12$, $AB = 13$. L, M and N, are respective midpoints of AC, BC and AB. Find the length of median MC. $[\frac{13}{2}]$

A3. Expand $(\frac{2}{3}x - \frac{y}{4})^3$. $[\frac{8x^3}{27} - \frac{x^2y}{3} + \frac{xy^2}{8} - \frac{y^3}{64}]$

A4. How many three-digit even numbers can be formed from the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, if repetition of digits is not allowed? [252]

A5. A set of 9 numbers are arranged from smallest to highest. The average of the first 5 numbers is 7, while the average of the last 5 numbers is 12. If the average of all 9 numbers is $8\frac{2}{9}$, find the median of the set. [21]

D1. In Jose Rizal High School, there are 1500 students. Every day, each student takes 6 classes, and each teacher teaches 5 classes. Each class has 40 students and 1 teacher. How many teachers are there at Jose Rizal High School? [45 teachers]

D2. In a classroom, two-fifths of students are wearing black socks, and three-fourths of the students are wearing gray pants. What is the minimum number of students in the classroom wearing both black socks and gray pants? [3]

D3. The vertices of a triangle are $(0, 0)$, $(3, 5)$, and $(6, 2)$. Find the coordinates of the centroid of this triangle.
$$[(3, \tfrac{7}{3})]$$

D4. Find the radius of the circle inscribed in an isosceles triangle with base 8 and legs 12.
$$[2\sqrt{2} \text{ units}] \ (\tfrac{8\sqrt{2}}{3} \text{ was said})$$

D5. How many triangles are in the given configuration? [84 triangles]

C1. Solve for x: $x^2 + 2015x = 2016$. $[1, -2016]$

C2. Solve for x: $x^{2016} - x^{2014} = x^{2015} - x^{2013}$. $[-1, 0, 1]$

C3. What is the area of the region bounded by the graph of $|x + y| + |x - y| = 4$?
[16 sq. units]

Grade 9.

E1. A triangle has many "centers". How do we call the center of the triangle formed by its three angle bisectors?
[Incenter]

E2. In simplest form, what is the numerical value of $\sqrt{2016}\sqrt[3]{2016}\sqrt[6]{2016}$? [2016]

E3. What is the value of $\sin^{-1} -\tfrac{1}{2}$? $[-\tfrac{\pi}{6} \text{ or } -30°]$

E4. What is $\cos(\sin^{-1} \tfrac{1}{3})$? $[\tfrac{2\sqrt{2}}{3}]$

E5. Simplify $\cos^{-1}\theta + \sin^{-1}\theta$. $[\tfrac{\pi}{2} \text{ or } 90°]$

E6. There are 2 each of red, green, blue and white socks. What is the minimum number of socks to pull out to get a matching pair? [5]

E7. How many three-letter words can be formed by the letters of LOGARITHMS without repetition? [720]

E8. Evaluate $2016 \times 20152014 - 2015 \times 20162015$. $[-1]$

E9. Find 4 consecutive odd integers whose sum is 2016. [501, 503, 505, 507]

E10. If $4^a = 5$, $5^b = 6$, $6^c = 7$, and $7^d = 8$, find the value of $abcd$. $[\tfrac{3}{2}]$

A1. Find the infinite sum $1 + \sqrt{5} + 5\sqrt{5} + 25 + 25\sqrt{5} + 125 + \ldots$ [no sum]

A2. Find $\sin 75°$. $[\tfrac{\sqrt{2}+\sqrt{6}}{4}]$

A3. Find the numerical coefficient of x^{10} in the expansion of $(x - 1)^{15}$. $[-3003]$

A4. The point with coordinates $(1000, 2016)$ is reflected on the line $y = 2000$. What are the coordinates of this point? $[(1000, 1984)]$

A5. Solve for x: $|x - 2|(x - 4)(x + 5) \geq 0$. $[x \geq 4, x = 2, x \leq -5]$

D1. Find the sum of all integer solutions of $1 < (x - 2)^2 < 25$ [12]

D2. If a and b are one-digit positive integers, find all the fractions $\tfrac{a}{b}$ where the infinite series $\tfrac{a}{b} + \tfrac{a}{b^2} + \tfrac{a}{b^3} + \cdots = \tfrac{1}{3}$.
$$[\tfrac{1}{4} \text{ and } \tfrac{2}{7}]$$

D3. In triangle ABC, $AB = 2AC$. Point D and R are on segments AB and AC respectively such that $\angle BAC = \angle ACD$. Find the measure of angle A. $[90°]$

D4. In a 13-14-15 triangle, find the length of the altitude to the base of 14 units. [12 units]

D5. In triangle ABC, $a = 7$ cm, $b = 3$ cm and $c = 5$ cm. Find the measure of $\angle A$. $\hfill [120°]$

Grade 10.

E1. If $x = 2.3 \times 10^{2015}$ and $y = 7.7 \times 10^{2016}$, what is $x + y$ in scientific notation? $\hfill [7.93 \times 10^{2016}]$

E2. In how many ways can we select a group of 3 men and 4 women from a group of 6 men and 5 women?
$\hfill [100 \text{ ways}]$

E3. A circle has center at $(-3, 6)$ and passes through $(3, 3)$, find its radius. $\hfill [3\sqrt{5} \text{ units}]$

E4. In triangle ABC, $\angle A = 60°$, $AC = 2$ cm and $AB = 3$ cm, what is BC? $\hfill [\sqrt{7} \text{ cm}]$

E5. Factor the expression $x^2 + 4x - 1$ in the field of real numbers. $\hfill [(x + 2 - \sqrt{5})(x + 2 + \sqrt{5})]$

E6. At least how many coins is to be tossed on a table so that the probability of getting all heads is less than $\frac{3}{100}$? $\hfill [6]$

E7. What is the maximum value of $f(x) = (2 - x)(x + 6)$? $\hfill [16]$

E8. If $y = 2x$ and $x + 2y = 15$, what is y? $\hfill [6]$

E9. The legs of a right triangle are 6 and 8 cm. What is the area of the circle that circumscribes the right triangle?
$\hfill [25\pi \text{ sq. cm}]$

E10. If the vertex of the parabola $y = 2x^2 + bx + c$ is $(2, 3)$, what is c? $\hfill [11]$

A1. If $3, a, b, c, 27$ form a geometric sequence, what are the possible values of $a + b + c$? $\quad [9 + 12\sqrt{3},\ 9 - 12\sqrt{3}]$

A2. A man is 8 years older than his wife, and 6 times as old as his son. When the son was born, the age of the wife is four-fifth of that of his husband. Find the age of the son now. $\hfill [8 \text{ years old}]$

A3. If -1 is one of the roots of $2x^3 - 3x^2 - 8x + k = 0$, find the other roots. $\hfill [-\frac{1}{2}, 3]$

A4. What is the area of a regular hexagon with perimeter 48 cm? $\hfill [96\sqrt{3} \text{ sq. cm}]$

A5. Find the tenth term of a non-constant arithmetic sequence, where the first term is 4, and whose first, fifth, and seventeenth terms form a geometric sequence. $\hfill [22]$

D1. Simplify $1 + \cfrac{1}{1 - \cfrac{1}{1 + \cfrac{1}{x}}}$. $\hfill [x + 2]$

D2. Find the solution set of $x^3 + 12 \leq x^2 + 8x$. $\hfill [(-\infty, 3] \cup 2]$

D3. What are the coordinates of the point on $(x - 1)^2 + (y - 2)^2 = 5$ that is closest to the point $(7, -1)$? $\quad [(3, 1)]$

D4. Charles has a probability of $\frac{1}{3}$ in winning a game against his younger sister. If they play four times, what is the probability that he will win at least twice? $\hfill [\frac{11}{27}]$

D5. If $a_1 = 3$, $a_2 = 5$, and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 3$, what is a_{2016}? $\hfill [1 + 2^{2016}]$