

MMC 2017

Regional Level, Category B

March 29, 2017

I can vouch for accuracy as I was able to attend the round. If you have any corrections or additions, please contact me at cj@cjquines.com, or through my Facebook account, Carl Joshua Quines.

Grade 7

- E1. Find the value of $\frac{30}{6}(6^2 - 3^3)$. [45]
- E2. If A is the set of all multiples of 3, and B is the set of all composite numbers less than 10, find $A^C \cap B$. [4, 8]
- E3. A rectangular garden $27 \text{ m} \times 12 \text{ m}$ is surrounded by a walk 1.5 m wide. What is the area of the walk? [126 m²]
- E4. Simplify: $\sqrt{150} + \sqrt{175} - \sqrt{54} + \sqrt{63}$. [$8\sqrt{7} + 2\sqrt{6}$]
- E5. Two exterior angles of a quadrilateral sum up to 200° . What is the measure of the other two exterior angles if they are equal? [80°]
- E6. A stove offered at a 10% discount is now 1350 pesos. What is its original price? [1500 pesos]
- E7. What is the sum of the first 200 positive even numbers?¹ [40200]
- E8. Evaluate $(a + b)^{ab}$ if $a = \frac{1}{3}$ and $b = 6$. [$\frac{361}{9}$]
- E9. How many liters is 75 cm^3 ? [0.075 L]
- E10. The mean of a set of four numbers is 10. If the numbers in the set are increased by 1, 2, 3, and 4, respectively, what is the new mean? [12.5]
- A1. What is the $\sqrt{119}$ to the nearest hundredth? [10.91]
- A2. What should be added to $\frac{3}{11}$ to get $-\frac{11}{3}$? [$-\frac{130}{33}$]
- A3. What should be the value of a if $ax^2 - 2x + 3a = -25$ when $x = 6$? [$-\frac{1}{3}$]
- A4. Simplify: $\frac{12^5 \times 4^3 \times 9^2}{6^2 \times 8^3 \times 54}$. [1296]
- A5. The shortest side of a triangle is half the longest. The third side is five more than the shortest. The perimeter is between 32 and 80, exclusive. What could the longest side be, in interval notation? [$(\frac{27}{2}, \frac{75}{2})$]
- D1. Solve the inequality $\left| \frac{2x + 3}{5} \right| - \frac{5x + 1}{2} \leq 1$. [$[-\frac{3}{7}, \frac{11}{21}]$]
- D2. A square is inscribed in a circle, and its diagonal is 16 cm. Find the area of the region between the circle and square. Use $\pi = 3.14$. [72.96 cm²]

¹Original answer 10100 was contested by pretty much all the teams.

- D3. There are two data values in class 11 to 16, five in class 17 to 22, and three in class 23 to 28. What is the mean? [20.1]
- D4. If the number of sides of a polygon were tripled, the sum of the interior angles would be quadrupled. What is the sum of the interior angles of the original polygon? [720°]
- D5. Points A, B, C , and D are collinear, and $AB = 15, AC = 10, BC = 5$. If $CD = 7$, what can AD be? [3, 17]

Grade 8

- E1. If $f(x) = \frac{3x + 1}{2x^2 - 4}$, compute $f(-2)$. $\left[-\frac{5}{4} \right]$
- E2. How many digits does the quotient $\frac{1.5 \times 10^6}{3 \times 10^{-1}}$ have? [7 digits]
- E3. Give an equation in slope-intercept form of the line passing through $(-1, 8)$ and with x -intercept of 3. $[y = -2x + 6]$
- E4. For which value/s of k is the system $-3x + 4y = 5$ and $k^2x - 3y = 8$ inconsistent? $\left[-\frac{3}{2}, \frac{3}{2} \right]$
- E5. What is the value of $(3^{-1} - 1)^{-3}$? $\left[-\frac{27}{8} \right]$
- E6. The square of Fat's is 279 more than the square of Roy's age. If Fat is 9 years older than Roy, what is the sum of their ages? [31]
- E7. What is the area of the triangle formed by the line $y = \frac{3x + 6}{-2}$ with the coordinate axes? [4 sq. units]
- E8. A dice is loaded so that the probability of a number coming out is directly proportional to the number. When the die is rolled, what is the probability that a prime number comes out? $\left[\frac{10}{21} \right]$
- E9. At least how many children must a family have so that the probability of having at least one daughter is more than 95%? [5 children]
- E10. What is the difference between the sum of the first 500 positive multiples of three and the first 500 positive integers? [250500]
- A1. Simplify the expression so that there are no negative exponents. $\frac{(6a^2b^{-2}c)^3}{(12a^3bc^{-3})^2}$. $\left[\frac{3c^9}{2b^8} \right]$
- A2. The mean of nine numbers is 86. If two of the numbers, namely, 90 and 96, were removed, what would be the mean of the remaining numbers? [84]
- A3. Simplify $2 - \frac{2x^2 - 5x - 3}{x^2 - 6x + 9}$. $\left[-\frac{7}{x - 3} \right]$
- A4. Give the solution set to the system $4x - 14y = -4$ and $10x - 35y = -10$. $[\{(x, y) | 2x - 7y = -2\}]$
- A5. Compute $\frac{3}{2 \times 7} + \frac{3}{7 \times 12} + \frac{3}{12 \times 17} + \cdots + \frac{3}{2012 \times 2017}$. $\left[\frac{1209}{4034} \right]$
- D1. It takes Lucy 20 minutes longer than her sister to clean the bedroom. Working together, they can clean their bedroom in 24 minutes. How long does it take Lucy to clean the bedroom by herself? [60 minutes]

- D2. A square picture is to be mounted on a frame with a 2 in margin on each side. If the area of the border is 32 in^2 , what is the perimeter of the picture? [8 in]
- D3. If a cube has side length $(2 - \sqrt{3})$ cm, which is greater, and by how much: the volume of the cube, or its surface area? [surface area, by $16 - 9\sqrt{3}$]
- D4. Factor completely: $64x^6 - y^{12}$. [$(2x - y^2)(4x^2 + 2xy^2 + y^4)(2x + y^2)(4x^2 - 2xy^2 + y^4)$]
- D5. A rectangular beam is to be cut from a circular block of diameter 10 in. If the rectangular cross-section of the beam has area 40 in^2 , what are the dimensions of the cross-section? [$4\sqrt{5} \text{ in} \times 2\sqrt{5} \text{ in}$]
- C1. Two distinct integers are randomly chosen from 1 to 10. By how much is the probability that the sum of the two numbers is even greater than the probability that the sum is odd? [0]
- C2. If p is a prime number, and $p + 1$ is a perfect cube, give all possible values for p . [7]
- C3. How many positive integer solutions are there to $3x + 4y \leq 25$? [20]

Grade 9

- E1. Two similar polygons have areas in the ratio 5 : 4. The smaller one has perimeter 4 units. What is the perimeter of the larger polygon? [$2\sqrt{5}$ units]
- E2. Find the vertex of the graph of $y = x^2 + 2x - 3$. [(-1, -4)]
- E3. What is the largest root of $(x + 7)(x - 7) = 9(x + 7)$? [16]
- E4. Find the smallest integer x which satisfies $x^2 + x < 20$. [-4]
- E5. The legs of a right triangle are 5 cm and $\sqrt{3}$ cm long. How long is the median to the hypotenuse? [$\sqrt{7}$ cm]
- E6. What is the probability that an integer chosen from 1 to 20 inclusive is prime? [$\frac{2}{5}$]
- E7. A rhombus has area 40 cm^2 . Its two diagonals differ by 2 cm. How long is the shorter diagonal? [8 cm]
- E8. The two roots of $x^2 - bx + 213 = 0$ are positive prime numbers. Find b . [74]
- E9. The two opposite angles of a parallelogram have measures x° and $(3x - 20)^\circ$. Find x . [10]
- E10. In right triangle ABC , $\cos A = \frac{9}{11}$. Find $\tan A$. [$\frac{2\sqrt{10}}{9}$]
- A1. A rhombus has perimeter 60 units. If the diagonals have lengths $2x - 2$ and $2x + 4$, find x . [10]
- A2. If r and s are the roots of $x^2 - 3\sqrt{2}x + \sqrt[3]{4} = 0$, find $r^3 + s^3$. [36]
- A3. Suppose that z is proportional to x^2 and inversely proportional to y . If $x = \sqrt{7}$ and $y = \sqrt{11}$, then $z = 13$. Find z if x is doubled and y is halved. [104]
- A4. A box contains 3 black balls, 4 white balls, and 6 red balls. Find the smallest number of red balls that must be added so that the probability of picking a red ball from the box is at least $\frac{3}{5}$. [5]
- A5. In a triangle, the two sides adjacent to its 60° angle have lengths 2 and 4 units. Find the length of the third side. [$2\sqrt{3}$ units]

²This was read, attempted to be contested, and failed. I still have no idea what happened.

- D1. Find the range of values of the constant k so that the quadratic equation $16x^2 - 8\sqrt{3}(k-2)x + k + 2 = 0$ has no real root. $\left[\left(1, \frac{10}{3} \right) \right]$
- D2. If the sum of the radius of a circle and the side length of a square is to be fixed at 2 units, what is the smallest possible sum, in square units, of the areas of the two figures? $\left[\frac{4\pi}{\pi + 1} \right]$
- D3. In parallelogram $ABCD$, points E and F are chosen on sides AB and CD , respectively, so that $AE = DE$ and $\frac{CF}{DF} = \frac{2}{3}$. Find the ratio of the area of triangle BFC to the area of quadrilateral $BEDF$. $[4 : 11]$
- D4. List down all the solutions of $3 \left(x + \frac{1}{x} \right)^2 - 16 \left(x + \frac{1}{x} \right) + 20 = 0$. $\left[\frac{1}{3}, 1, 3 \right]$
- D5. In a rectangle $ABCD$, P is chosen on AB and R on CD so that $AP = 9$ and $DR = 15$. Point Q is chosen on BC such that $\angle PQR = 90^\circ$. If $PQ = 4\sqrt{6}$ and $QR = 2$, find the area, in square units, of pentagon $APQRD$. $[96 + 4\sqrt{6}]$