

# MMC 2017

Sectoral Level, Category B

March 24, 2017

Disclaimer: inexact wording. I was not able to attend the actual round, so I must rely on transcriptions, which may affect the accuracy. I can vouch for grades 7 and 8, as I was able to listen to a recording; I had to rely on two incomplete transcriptions for grades 9 and 10. With thanks to Sean Anderson Ty, Jireh Gumaro, John Michael Rivera, and Nathanael Balete for sending corrections. If you have any corrections or additions, please contact me at [cj@cjquines.com](mailto:cj@cjquines.com), or through my Facebook account, Carl Joshua Quines.

## Grade 7.

- E1. The following scores are in ascending order: 10, 12, 16, 19, 20 and 22. Find the median. [17.5]
- E2. Convert  $250 \text{ cm}^2$  to  $\text{m}^2$  in fraction form.  $\left[\frac{1}{40}\right]$
- E3. Compute  $\frac{6(-3 + 12)}{-2} - 8$ . [-35]
- E4. Twice the measure of angle  $A$  plus  $30^\circ$  is equal to its complement. Find the measure of angle  $A$ . [ $20^\circ$ ]
- E5. What integer is closest to  $\sqrt{450}$ ? [21]
- E6. Convert  $122^\circ\text{F}$  into  $^\circ\text{C}$ . [ $50^\circ\text{C}$ ]
- E7. Among all rectangles with integral sides whose area is  $100 \text{ m}^2$ , what is the largest perimeter? [202 m]
- E8. The measure of angle  $AOB$  is equal to  $120^\circ$  and the measure of angle  $AOC$  is equal to  $40^\circ$ . What are the possible measures of angle  $COD$ ? [ $80^\circ, 160^\circ$ ]
- E9. Solve the equation  $|3x - 1| = 5$ .  $\left[-\frac{4}{3}, 2\right]$
- E10. The length of a rectangular garden is 4 m longer than the width. If the perimeter is 52 m, what is the area? [ $165 \text{ m}^2$ ]
- A1. In a class of 50 students, 17 read Noli Me Tangere, 13 watched its play, and 10 watched its TV series. Also, 8 both read the local and watch the play, 4 watch the play and the TV series, and 6 read the novel and watched the TV series. Nobody did all three. How many students had not done any of the three activities? [28]
- A2. How much greater is  $5x^3 - x^2 + \frac{7}{2}$  than  $\frac{3}{2}x^3 + 7x - 5$ ?  $\left[\frac{7}{2}x^3 - x^2 - 7x + \frac{17}{2}\right]$
- A3. How many grams, to the nearest integer, is 20 pounds? [40909]
- A4. Jinky drove a distance of  $x^3y^2$  kilometers for  $xy^3z^2$  hours. What is her average speed?  $\left[\frac{x^2}{yz^2} \text{ kph}\right]$

- A5. Vangie's average score in four exams is 87. If her teacher decides to count the fourth exam double, and Vangie's score in that exam is 97, what will Vangie's new average be? [89]
- D1. If  $\frac{x}{5} = \frac{y}{3} = \frac{z}{6}$  and  $xyz = 2430$ , find  $x + y + z$ . [42]
- D2. The volume of a box is  $6x^3 - 17x^2 - 43x + 120$ . If its length is  $2x - 5$  and its width is  $x - 3$ , find its height. [ $3x + 8$ ]
- D3. In rectangle  $PQRS$ , points  $M$  and  $N$  are midpoints of  $PQ$  and  $QR$ , respectively. If the area of triangle  $MNS$  is 63, find the area of triangle  $NSR$ . [42 sq. units]
- D4. Estimate  $\sqrt{97}$  to the nearest hundredth. [9.85]
- D5. The denominator of a rational number is 7 more than its numerator. If 16 is added to the numerator and 15 is subtracted from the denominator, the result is  $\frac{5}{2}$ . What is the rational number? [ $\frac{24}{31}$ ]

### Grade 8.

- E1. If  $f(x) = \frac{9x^2 - 25}{3x - 5}$ , compute  $f(-6)$ . [-13]
- E2. If the volume of a cube is 20% more than its surface area, what is the side length of the cube? [ $\frac{36}{5}$  units]
- E3. How many digits does the quotient  $\frac{2.5 \times 10^{-4}}{5 \times 10^{-2}}$  have?<sup>1</sup> [6]
- E4. What is the cube root of  $6.4 \times 10^{-8}$ ? [ $4 \times 10^{-3}$ ]
- E5. If two dice are rolled, what is the probability that the product of the two numbers that come out is even? [ $\frac{3}{4}$ ]
- E6. Compute the product  $127 \times 133$ . [16891]
- E7. Simplify:  $\frac{185^2 - 75^2}{11}$ . [2600]
- E8. If the sum of two prime numbers is 1053, what is their product? [2102]
- E9. What integer greater than 1 is always a divisor of the difference between the squares of two odd integers?<sup>2</sup> [2, 4 or 8]
- E10. In a class of 30 students, 21 have at least one sister and 15 have at least one brother. At most how many students have no siblings? [9]
- E11. Give the exact value of  $\frac{3\sqrt{3} + 125}{\sqrt{3} + 5}$ . [ $28 - 5\sqrt{3}$ ]
- A1. Simplify so that there are no negative exponents:  $\left(-\frac{24a^4b^{-1}}{45a^{-3}b^{-6}}\right)^{-2}$ . [ $\frac{225}{64a^{14}b^{10}}$ ]

<sup>1</sup>This is how the question was read at the public sectorals, where it was voided due to unclear use of "digits" for decimals. The private sectorals has  $2.5 \times 10^4$  as its numerator.

<sup>2</sup>This is how the question was read at the public sectorals. The original answer was 4, which was corrected. In the private sectorals, the question was read as "...integer greater than 1 is *the greatest common divisor* of the...", with correct answer 8.

- A2. The mean of five numbers is doubled if one of the numbers is increased by  $x$ . What is the mean of the original five numbers?<sup>3</sup>  $\left[\frac{x}{5}\right]$
- A3. Two sides of a triangle have lengths 17 and 22. What is the length of the third side if it is a multiple of 9 and a sum of two squares?  $[18]$
- A4. If  $x = \sqrt{3} + 1$  and  $y = \sqrt{3} - 1$ , compute  $x^4 - y^4$ .  $[32\sqrt{3}]$
- A5. Factor:  $x^3 - 2x^2 + 1$ .  $[(x - 1)(x^2 - x - 1)]$
- A6. How many liters of a 90% acid solution must be added to 4 liters of 70% solution to obtain a mixture with 75% acid?  $\left[\frac{4}{3}\right]$
- D1. If  $(3, 2)$  is the solution to the system  $(a - 1)x - by = 7$  and  $2bx + (a + 1)y = 16$ , what are the values of  $a$  and  $b$ ?  $[a = 4, b = 1]$
- D2. Solve for  $x$ :  $10(3^x) - 9^{x+1} = 1$ .  $[x = -2, 0]$
- D3. For which integers  $n$  is  $n^2 - 6n - 27$  a prime number?  $[n = -4, 10]$
- D4. What values of  $x$  satisfy the equation  $\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$ ?  $[x \in (-\infty, -1) \cup (-1, 1) \cup (1, \infty)]$
- D5. Triangle  $ABC$  is a right triangle with hypotenuse  $AC = 15$ . It shares a common side with square  $ABED$ . If  $BC$  has an integer length, and the area of  $ABED$  is prime, what is the area of  $ABED$ ?  $[29]$

### Grade 9.

- E1. Solve for  $x$  in the quadratic equation  $(x + 1)^2 = 3(x + 1)$ .  $[x = -1, 2]$
- E2. Find the vertex of the graph of  $y = x^2 - 5x$ .  $\left[\left(\frac{5}{2}, -\frac{25}{4}\right)\right]$
- E3. Two adjacent angles of a parallelogram differ by  $40^\circ$ . Find the smaller angle.  $[70^\circ]$
- E4. Find the largest positive integer in the solution set of  $x^2 + x - 12 \leq 0$ .  $[3]$
- E5. What is the sum of the roots of  $2017x^2 + 4034x + 1 = 0$ ?  $[-2]$
- E6. If  $a : b = 3 : 4$  and  $b : c = 3 : 4$ , find the ratio  $a : c$ .  $[9 : 16]$
- E7. Simplify the expression  $\sqrt[3]{27x^3y^4z^5}$ .  $[3xyz\sqrt[3]{yz^2}]$
- E8. The diagonals of an isosceles trapezoid have lengths  $x^2 - 3x$  units and 10 units. Find  $x$ .  $[x = -2, 5]$
- E9. An urn contains 5 white balls, 5 black balls, and 5 red balls. Each group of balls with the same color is numbered from 1 to 5. Find the probability that a ball picked is white and contains a prime number.  $\left[\frac{1}{5}\right]$
- E10. The three sides of a right triangle are consecutive even integers. Find its area in square units.  $[24 \text{ sq. units}]$
- A1. Suppose that  $z$  is proportional to  $x$  and inversely proportional to  $y$ . If  $x$  is 6 and  $y$  is 4, then  $z$  equals 3. If  $x$  equals 4 and  $y$  equals 6, find  $z$ .  $\left[z = \frac{4}{3}\right]$

<sup>3</sup>Voided at private sectorals due to unclear phrasing. The question was read as "... is doubled. If one of the numbers is increased by  $x$ , what is the...".

- A2. Rationalize the denominator of  $\frac{\sqrt{3} + 16}{2\sqrt{3} - 1}$  and simplify.<sup>4</sup> [2 + 3\sqrt{3}]
- A3. In triangle  $ABC$ ,  $D$  and  $E$  are the midpoints of  $AB$  and  $AC$ , respectively. If  $DE$  is  $2x + 4$ ,  $BC$  is  $7x - 1$ ,  $AE$  is  $4x$  and  $EC = 3y$ , find  $x$  and  $y$ . [ $x = 3, y = 4$ ]
- A4. If  $r$  and  $s$  are the roots of  $x^2 - 4x + 2\sqrt{3} = 0$ , find  $\frac{1}{r^2} + \frac{1}{s^2}$ . [ $\frac{4 - \sqrt{3}}{3}$ ]
- A5. Two angles of a triangle are  $45^\circ$  and  $75^\circ$ . If the side between them has length  $\sqrt{6}$ , find the length of the shortest side. [2 units]
- A6. Simplify  $5\sqrt{20} + \frac{2}{5}\sqrt{45} - \sqrt{\frac{1}{5}}$ . [11\sqrt{5}]
- D1. Solve for  $x$  in the equation  $4 + \sqrt{x} = \sqrt{4 + 5x}$ . [ $x = 9$ ]
- D2. The sides of a triangle have lengths 5, 6, and 7. What is the length of the altitude perpendicular to the side with length 6? [ $2\sqrt{6}$  units]
- D3. Solve for  $x$  in the equation  $(x^2 + 3x)^2 - 14(x^2 + 3x) + 40 = 0$ . [ $x = -5, -4, 1, 2$ ]
- D4. Of the people gathered in a room, 5 do not use any of the brands  $X, Y$  and  $Z$ , and 2 people use all three. It turns out that that 7 use  $X$  and  $Y$ , 5 use  $X$  and  $Z$ , and 9 use  $Y$  and  $Z$ . Furthermore, 9 use  $X$  but not  $Z$ , 13 use  $Y$  but not  $X$ , and 11 use  $Z$  but not  $Y$ . Find the probability that a person picked from this room uses exactly one brand. [ $\frac{9}{20}$ ]
- D5. The two bases of a trapezoid have lengths  $6x$  (the shorter base) and  $4x^2 + 4x$  units. Its two diagonals intersect the line connecting the midpoints of the sides (the midline), at points  $A$  and  $B$ . If  $AB = 3$ , find  $x$ . [ $\frac{3}{2}$ ]
- C1. Find all values of the constant  $c$  that will make  $x^2 + cx + \frac{49}{4}$  a perfect square trinomial.<sup>5</sup> [ $c = -7, 7$ ]
- C2. The longest sides of three similar triangles are 3, 4 and 5 units. If the sum of their areas is 200 square units, find the area in square units of the smallest triangle. [36 units<sup>2</sup>]
- C3. A rectangular pen needs fencing for only three sides. The fencing for two parallel sides costs 75 pesos per meter, while the fencing for the third side costs 100 pesos per meter. If the total budget for fencing is 3000 pesos, how long should each of the two parallel sides be to maximize the area? [10 m]

### Grade 10.

- E1. From 6:10 PM to 7:45 PM of the same day, how many degrees does the minute hand rotate? [570°]
- E2. What is the remainder when  $3x^6 - 10x^2$  is divided by  $x - \sqrt{2}$ ? [4]
- E3. What is the radius of the circle  $x^2 + y^2 + 4x - 6y = 3$ ? [4]
- E4. Four times a certain number is 20 units below 10. What is the number? [ $-\frac{5}{2}$ ]
- E5. The minimum value of  $f(x) = x^2 + 5x + b$  is 0. What is  $b$ ? [ $\frac{25}{4}$ ]

<sup>4</sup>Voided at private sectorals, because the reading of the fraction was unclear.

<sup>5</sup>The clincher questions were read for private sectorals.

- E6. If  $f(x) = x^2 - 3x$ , what is  $f(-1) - f(1)$ ? [6]
- E7. Triangle  $ABC$  is inscribed in a circle with center at point  $O$ . If angle  $AOB$  is  $48^\circ$ , find angle  $ACB$ . [ $24^\circ$ ]
- E8. If a 64 cm stick is cut into three parts in the ratio  $2 : 4 : 6$ , how long is the longest part? [32 cm]
- E9. A variable  $p$  varies inversely as the square root of  $q$ . If  $q$  is 81 when  $p$  is 4, what is the constant of proportionality? [36]
- E10. There are two concentric circles. A chord of length 10 cm of the larger circle is tangent to the smaller circle. What is the area between the two circles? [ $25\pi \text{ cm}^2$ ]
- A1. Find the sum of all positive integers less than 45 which are not divisible by 3. [675]
- A2. The minimum value of  $f(x) = 3x^2 - 4x + b + 2$  is 0. What is  $b$ ? [ $-\frac{2}{3}$ ]
- A3. The second and thirtieth terms of an arithmetic sequence are 7 and 91, respectively. What is the tenth term of the sequence? [31]
- A4. What is the smallest integer  $k$  such that  $2x(kx - 4) - x^2 + 6 = 0$  has no real solutions? [2]
- A5. Given  $f(x) = 5x^2 - 2x - 1$ , what is  $f(x + h) - f(x - 2h)$ ? [ $30hx - 6h - 15h^2$ ]
- D1. By selling balloons at 12 pesos each, a vendor gains 19.7%. The cost price of balloons rises by 12.5%. If he sells the balloons at the same price as before, what is his new gain in percent?<sup>6</sup> [6.4%]
- D2. How long is the shortest chord that can be drawn through a point 20 cm from the center of a circle whose radius is 29 cm long? [42 cm]
- D3. A father is three times as old as his son. Four years ago, he was four times as old as his son. How old was the son ten years ago? [2 years old]
- D4. A regular hexagon and an equilateral triangle have equal perimeters. If the area of the hexagon is  $6\sqrt{3} \text{ cm}^2$ , what is the area of the triangle? [ $4\sqrt{3} \text{ cm}^2$ ]
- D5. An isosceles trapezoid is circumscribed about a circle. If the bases are 18 cm and 6 cm, what is the radius of the circle? [ $3\sqrt{3} \text{ cm}$ ]

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<sup>6</sup>Some claim that the reader for the public sectorals said balut instead of balloons. This does not affect the problem.