

PMO 2016 Reflections

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These are my personal thoughts regarding the conduct of the 18th Philippine Mathematical Olympiad. I have written this mostly for the sake of my personal reflection and for archival purposes. As such, this is a very personal document.

A word of warning: this report is 23 pages long, documenting an event which happened, for the most part, over three days. The sheer number of words put in¹ is daunting compared to other reports. The ratio of the number of words to the number of events that happened is about thrice as large as any other report I've read. If I join and consequently write a report about the IMO I might end up with a novella.²

1 Introduction

For the benefit of those whom have never heard of the PMO before:

The Philippine Mathematical Olympiad is the oldest and most prestigious mathematical competition in the Philippines. This year, 2016, marks the 18th year of its conduct from its beginning in 1984.

The PMO is open for entry to all Philippine high school students. There are three stages in the competition, the qualifying stage, the area stage and the national stage.³

The qualifying stage is a two-hour written examination consisting of 25 multiple-choice questions followed by five short-answer problems. The top 50 scorers of each area (Luzon, Visayas, Mindanao and NCR) proceed to the area stage, a three-hour written examination consisting of 15 short-answer problems and three open-ended problems which require solutions. Finally, the top 20 scorers from the whole country in the area stage proceed to the national stage.

The national stage consists of a closed-door written phase and an oral phase which is open to the public. The written phase is a four-and-a-half-hour examination consisting of five problems requiring a full written solution, worth eight points each. The oral phase consists of 30 questions being read to the participants, 15 two-point easy questions to be answered in 15 seconds, 10 three-point medium questions to be answered in 30 seconds and 5 six-point hard questions to be answered in 60 seconds. Difficulty is of course relative, all the questions are challenging.

The ranks from both written phase and oral phase are combined, 70% coming from the written phase and 30% coming from the oral phase, to form the final ranking. The top three contestants are awarded trophies, medals and money.

¹Approximately 11,000 words. For comparison, Gabriel Gendler's 2014 IMO report, which reports on something that happened over a fortnight rather than three days, is about 7,000 words long.

²... which is more than enough reason to include me as a part of the Philippine team! Please?

³This is similar, for example, to USA's AMC, AIME and USAMO. The PMO, however, is far easier. . .

The 20 contestants who qualified for the national stage are then trained over the summer in the Mathematical Olympiad Summer Camp, a five-phase intensive training program to select the six-member Philippine team for the International Mathematical Olympiad.

This year, 4622 high school students participated in the qualifying round, an all-time high for the PMO, with 224 making it to the area stage. Of these 224 students, 20 made it to the national stage.

2 Qualifying Stage

2.1 Pre-examination

The qualifying stage was held on October 17, 2015. At NCR, the testing venue was the Ateneo De Manila University, Quezon City, as it has been for a few years now. Our school, the Valenzuela City School of Mathematics and Science, sent five representatives for this round. They are, from most senior:

- Carl Joshua Quines (me), grade 10. I have only joined the PMO once before this, when I was grade 8, and I have only made it up to the qualifying stage. I was not able to participate last year, as I was out of the country during the qualifying stage.
- Axirazel Lorenzo, grade 10. This was the second year he joined the PMO, and the last year he only made it to the qualifying stage.
- Jireh Emmanuel Gumaro, grade 9. This was also the second year he joined the PMO, and the last year he also only made it to the qualifying stage.
- Mark Vincent Carabbay, grade 8. This was his first time to join the PMO.
- Vincent Dela Cruz, grade 7. This was also his first time to join the PMO.

We were accompanied to the competition by Marilyn Soriano, our school's mathematics coordinator. Our trainer was Augusto Logronio.

On the day of the competition we met each other at the school grounds, then we went to Ateneo. After registering and confirming that the staff got our names and our school's name were right (you won't believe how many times our school's name gets spelled wrong), we got our testing permits and went our separate ways. The five of us were all placed in different rooms. I was the only one whose room was on the fourth floor, while all their rooms were at the third floor, so they teased me about being lonely and whatnot.

I went to my assigned room. It was air-conditioned, which is always good for math. It is well known among mathematics competitors that an air-conditioned room is an essential part of every mathematics competition, and the PMO team delivered. There were 25 of us assigned to that room, and I got to my assigned room 30 minutes early, so I sat down patiently while waiting for the other contestants.

Whether or not sitting patiently involves going out of the room ever so often to take a bathroom break, or making really quiet noises by rubbing my hands or by breathing out noisily in an attempt to break the silence, I waited for the examination to start. Yes, people gave me weird looks for making strange noises, but whatever.

The proctor explained the conditions of the examination, no calculators, graphing paper, protractors, no bathroom breaks or food, and so on, the usual rules. Then I took a last-minute trip to the bathroom before the two-hour examination.

2.2 Examination proper

The full list of questions is available at the PMO website, so I will not list them again here. At the time of writing, the answers have not been posted yet, however, I do anticipate that they will be put up within the coming weeks.

Part I was relatively straightforward. After breezing past the first ten questions, I get stuck at number 11 because my answer was not a part of the choices. I decided to get back to number 11 later and answer the rest of Part I.

Part II was not as easy as Part I, of course, by sheer virtue of being Part II which should be more difficult than Part I. I had no clue how to solve number 5 at the time, and decided to get back to it later. I solved number 8 multiple times and each time my answer was not a part of the choices, so I marked the question as stupid and then proceeded to part III.

Part III, on the other hand, was far less insane. Number 1 was a relatively simple geometry problem straight from a My First Geometry Book. Number 2 was a functional equation which I remember not being able to solve at the time, but a linear function seemed to fit the conditions, despite me not having proof of it. So I assumed it was linear and went with that. Number 3 felt more fitting to be a part of Part I instead (I remember trying to get a system of equations, and then I realized I was stupid). Number 4 was a combinatorics problem of placing indistinguishable balls in distinguishable urns, but written in algebra to disguise its true nature. Finally, number 5 was an algebra question asking for the number of terms after simplifying a given expression.

None of the problems were especially original or remarkable. I did enjoy solving Part II number 1, Part III number 2 and Part III number 3. Difficulty-wise, the exam was about as difficult as past qualifying stages.

If perhaps I feel more motivated, I might do a write-up of the solutions to the qualifying stage as an exercise in writing in $\text{\LaTeX}$. But I do not feel motivated at the moment, alas. The PMO also does not release solutions to the qualifying stage.

2.3 Post-examination

The examination finished and I have been holding my pee in for thirty minutes. Dear reader: having to hold in your pee is bad for the concentration. I pass the examination to the front, and I caught a glimpse of the paper of the person behind me – we had the same answer for Part III number 3 so I felt slightly confident for at least that item. I stood up and, going out of the room, a girl asks me if I'm feeling well. Then I remembered all those weird sounds I made earlier, and reassure her that I do feel well, and that I just like weirding people out. I regret nothing.

Aside from that, there wasn't that much social interaction between the participants in the room. I suppose that was because they separated participants from the same school, so it was difficult to imagine that anyone in the room knew each other.

After rushing to the bathroom (which was closed on the floor we were on so I had to go down the stairs), I went down the building and waited for my schoolmates.

Then I remembered we didn't set a place to meet each other after the exam, which was stupid planning on our part. It took us something like ten minutes, and the halls were getting filled with people. Of course, with 800 students, it's difficult to find your schoolmates, especially if everyone's uniform is the same white color and your schoolmate has a shorter height than average. But we do find each other, and as soon as meeting each other we talk about the examination, as any self-respecting competitor should.

We see some graduates from our school at Ateneo, who studied there for college. There was Reggie Dela Cruz, whom proctored the exam of Vincent, and who was part of the team that won the MMC Oral Nationals last year. We also so Arzelis Herrera and Bret De Leon, both of whom were also mathematics competitors, and very successful students in terms of academic achievement. Then we rode home.

Throughout the entire ride home, as one would expect, we talked about the problems. The younger students realized they made stupid mistakes for some problems. We also talked about Part III number 4. I said my answer was 105, and Axirazel said that that seemed "too ugly to be correct". Of course it was too ugly to be correct – it isn't even a nice number. Better answers would be numbers like 120 or 36, which are far more beautiful and thus more likely to be an answer. But sadly, ugly answers are sometimes correct, like how $\frac{-1 \pm \sqrt{31}}{2}$ is sometimes correct even when you don't want it to be correct. The afternoon turned to night, and I arrived at my home, I ate my dinner and I went to sleep, thinking of what the future could possibly hold.

2.4 Aftermath

Two weeks later, on October 28, 2015, the questions for the qualifying stage were posted on the website, marking the questions as cleared for discussion in public. I went to a few Facebook groups and asked for help in the questions I didn't get, and I got enlightened by the solutions.

The week after that, on November 6, 2015, the qualifiers for the area stage were released on the PMO website. Ms. Soriano found out about the news first, and I only found out that I qualified when she told me.

When I found out about the news, I was of course ecstatic. I got a shot of qualifying to the national stage of the PMO, which was one of my goals. Of course, it was sad that the other contenders from our school didn't make it, but they were happy as well that I qualified for the area stage.

For the week between the announcement of qualifiers to the actual area stage, I looked at the list of my opponents. A number of them have made it to the nationals before. I looked at the contenders and thought that my chances of making it to the nationals were slim.

To think that out of 200 of us, only 20 would make it to the national stage meant that I had a 10% chance of making it. Of course, the chances were smaller because I could name a few people who were assured of making it to the national stage. And then again, the chances were slightly bigger from the fact that almost all of the competitors at the national stage are from my area, NCR.

But I stopped thinking about probabilities and instead focused on the area stage. I spent the little time I had preparing for the area, answering past PMO area stage questions.

3 Area Stage

3.1 Pre-examination

The area stage was held on November 14, 2015. At NCR, the venue was once again at the Ateneo De Manila University. From our school, only one person managed to make it past the qualifying stage to the area stage, Carl Joshua Quines (me).

I woke up that morning, ate my breakfast and went to school. Ms. Soriano accompanied me to Ateneo. We ate lunch out and arrived early to register. We then parted ways because she had to attend a class, so we designated a meeting point after the exam and she left.

I registered at the table and took a peek at the contestants for the area stage. Most of them were familiar names, but not familiar people, I wasn't friends with anyone else. I had a conversation with the people at the registration and at least they were pretty friendly.

We talked about not being afraid of going up against people better than us, which was pretty inspiring and uplifting, and we talked about Reggie, whom apparently managed to make his name known to everyone in the mathematics department after only being in school for a few weeks by asking his professor about the existence of ∞ in a class. Needless to say the professor proved so.

The person at the registration urged me to enter the room early to get acclimated to the lower temperature, and I did. Since there were only 50 students at the area stage, there were only two rooms for the examination, and both were on the first floor. I got assigned in the same room as Kyle Dulay and Farrell Wu, both names whom I recognized for winning lots of mathematics competitions. It was nice and air-conditioned, but not too cool for me to wear a jacket.

Everyone seemed like they knew each other. They all seemed like good friends with one another, and they talked with each other a lot. In contrast, I didn't know anyone else there. It was my first time to make it to the area stage of the PMO. I had never been to any of the sessions that the Mathematics Trainers Guild of the Philippines (MTG) held, or any mathematics camps or what, which was where I believed they met and got to know each other.

Before the examination began I tried to talk to the person sitting in front of me, Andres Rico Gonzales. He wasn't talking to anyone else at the moment, but when I asked him, he said he knew a few other people. I learned he studied at Letran and he was grade 8, but other than that, I didn't learn much about him, nor do I believe he learned much about me. But we talked quite a bit.

After a last-minute bathroom break, the three-hour examination began.

3.2 Examination proper

At the time of writing, a full list of questions is not available at the PMO website. In the event that the list of questions is not released when the national questions are, I will typeset and release the questions myself. If I can find my copy.

I took a look at the area stage paper. It, in fact, seemed way easier than the area stages of the past. It looked far, far simpler, and the questions looked a lot easier.

We were not provided with scratch paper and were told that we couldn't ask for any, and we couldn't use our own. Gasps of horror erupted from the entire room, as

everyone knew that scratch paper was essential for solving mathematics problems. The proctors told us to just use the back of the answer sheet as scratch paper.

A few minutes in and a few people noticed the typo at question 1. The question mentioned Marc and Mark, which apparently were the same person. We all laughed at the mistake once the proctor pointed it out and told us they were the same person.

Eventually the proctors came through with our request for scratch paper and they talked to the organizers, and finally we were given scratch paper. I didn't use the scratch paper much, because I worked out my solutions on the questionnaire itself.

Problems 2 through 5 were relatively easy, although I did make a mistake initially at problem 4 which I corrected later on in the examination. Problem 6 was a trigonometry problem, and it was fun and interesting and I liked it. Problem 7 was stupidly tedious, find the sum of all the prime factors of 27,000,001, and I only got $7 \times 43 \times 89701$. I tried bashing 89701 for factors and give up after trying all two-digit primes. Problem 8 was straightforward, and that was all the problems on the first page of the examination.

I go to the second page of the examination. Problems 9 to 12 were pretty straightforward. Problem 13 was a trigonometric system of equations which I hated, but I did manage to solve it. Problems 14 to 17 were also straightforward, and I used calculus to cheat at problem 18. Which is pretty much looked down upon when it comes to olympiads. All problems are designed to be solvable without using calculus.

Problem 19 was straightforward, and problem 20 was a bashy trial-and-error problem which I didn't like, though of course I knew that a solution which didn't involve as much bash that I did would appear somewhere or another.

Part II required solutions. The first problem was a number theory problem, and it was alright, and my solution only took a few lines. The second problem was an interesting algebra problem, and I enjoyed solving it, and it was fun, and my solution ended up using the whole page, even when I wrote with little letters. The third problem was a geometry proof which I killed using mass points. I briefly considered to use barycentric coordinates but then realized I knew too little about them to utilize them correctly. I tried thinking of a synthetic proof, but I thought for a minute and decided against it, and instead made better use of my time reviewing my answers for Part I.

I changed a few answers in Part I. After all my solving there was an hour left in the examination, which I decided to use by documenting at the back of my answer sheets. The back of the answer sheets, our proctor said, would not be read by the graders, so I decided to record the events that were happening at the moment.

I looked around. It looked like Kyle was finished, which I saw because he was sitting pretty close to me. I also saw a person or two whom I didn't know back then, and they were finished as well.

The last hour passed by too slowly. I had nothing to solve and I had nothing to do. Watching others was not interesting, as everyone was solving, and that wasn't interesting to watch. So I instead wrote a lot of words at the back of my answer sheet about my emotions and what I felt at the moment, which somehow turned out really depressing.

3.3 Post-examination

The examination finished and this time I wasn't holding in my pee for thirty minutes, so I didn't see a need to go to the bathroom immediately after the examination.

After the examination, the people whom knew each other quickly formed groups in

the room to talk about the area stage. Not being part of a group myself, I managed to overhear some conversations. It seemed like some people also thought that the area stage was easier compared to the past area stages.

I then talked to Andres about the area stage. We compared answers, and we got pretty similar answers for most questions. A few people whom I didn't know also got into our talk with Andres, and we talked about the problems as well.

The proctors told everyone to discuss the problems outside the room, because everyone all too quickly talked about the problems, and when high school students talk about math problems, they are quite difficult to stop. They did manage to get us all outside in a few minutes, which was certainly an achievement on their part.

Going outside the room, I overhear a few people talking about the frustrating problem 7, the infamous find the sum of the prime factors of 27,000,001. A lot of people also bashed 89701 up to the hundreds with no result. I believed 89701 to be a prime at the moment.

I meet up with Ms. Soriano and we took a ride home. We had dinner out, and during dinner I realized that I forgot the negative case in number 13, a trigonometric equation. We also talked about the acceptability of using mass points in a solution, and she told me not to worry because mass points is a pretty well-known technique and it was likely to be accepted in a proof. I made it home and went to sleep.

3.4 Aftermath

A few days after the examination, I tried seeing if 89701 had any factors through my computer. It turned out that 89701 was 271×331 , which marked my second mistake in Part I, the first being the trigonometric equation of number 13. I also verify my answer for problem 20, a problem looking for the minimum value of a four-digit number divided by the sum of its digits, by programming. It turned out I was right, the answer was $\frac{1099}{19}$, but I had no idea how anyone could get that without bashing.

I was browsing the Art of Problem Solving forum when I saw a post by Irrational.ph, a post about avoiding stupid mistakes. It turned out that he mentioned the PMO area stage in his post, and it turned out he took the area stage as well. We talked to each other and I learned that he was Sean Anderson Ty, a student from the Mindanao area.

We become friends on Facebook and compared our answers at the area stage. It also turned out that he didn't get problem 7, but that he knew someone from his school who managed to factor 89701. I realized I made a mistake on problem 10, missing the correct answer by one. We also made the same mistake on problem 13, forgetting the negative case. Finally, he made a mistake in problem 20. He was pretty confident for his answers in Part II, like I was. In total, we both anticipated that we made three mistakes in Part I and got Part II perfectly.

He believed that the area stage this year was also easier compared to the area stages of the previous years. We talk about possible cut-offs for the national stage, and he said that he already talked to other contenders. He believed we both had a good shot of making it to the national stage, as most people he talked to had more mistakes in Part I than we did. I, of course, was still worried immensely about not making it to the national stage. He said that his trainer already told him to prepare for nationals, so I decided that I would prepare for nationals a bit anyway, even though I believed my chances for getting in were bleak.

We talked to each other a lot on Facebook. We became friends because of math,

which I thought was neat because I had never gotten friends solely because of math before.

Almost every day of the month of December 2015 I checked the PMO website for the results of the area stage. The proctor said during the examination that results would be released on or before December 19, 2015, and I wondered how it would take a month to check 200 papers and announce the top 20. It turned out that the one day I didn't check for the results, the results were released.

On December 18, 2015, the national qualifiers and the winners of the area stage were released on the PMO website. I only found out about it the day after, on December 19. Once again, Ms. Soriano was faster on the news, as she was the one who told me that I qualified for the national stage.

I took a look at the PMO website. I saw the list of the twenty national finalists and I was ecstatic that I made the list. But what was even more exciting was the fact that I looked at area stage winners, and I found out that I won third place in the area stage, which was shocking and happy at the same time. Ms. Soriano and Mr. Logronio were of course very happy and excited by the news.

Sean also made it to the national stage, so that was happy, because that meant we would see each other on the national stage. A lot of people I knew by name only also made it to the national stage. Sean also won first in the area stage, which was awesome, so we congratulated each other.

The list of area stage winners follows. From Luzon,

1. Albert John Patupat, from Holy Rosary College.
2. Vince Jan Torres, from Santa Rosa Science and Technology High School.
3. Errol John Suarez, from Aquinas University of Legazpi Science High School.

From Visayas,

1. William Joshua King, from Bethany Christian School.
2. Myles Denzel Delatore, from Bethany Christian School.
2. Dominic Yap, from Philippine Science High School – Western Visayas.
3. Brigham Lucero, from Philippine Science High School – Central Visayas.

From Mindanao,

1. Xavier Jefferson Ray Go, from Zamboanga Chong Hua High School.
1. Sean Anderson Ty, from Zamboanga Chong Hua High School.
2. Vicente Raphael Chan, from Zamboanga Chong Hua High School.
3. Kenneth M. A. Antonio, from Bayugan National Comprehensive High School.

From NCR,

1. Clyde Wesley Ang, from Chiang Kai Shek College.
2. Luke Matthews Bernardo, from Philadelphia High School.

2. Kyle Patrick Dulay, from Philippine Science High School – Main.
3. Carl Joshua Quines (me), from Valenzuela City School of Mathematics and Science.

I was shocked to find out that I won third in the area stage. I expected Farrell Wu to be in the top three, and I was shocked to find out that somehow I did better than him in the area stage. Perhaps the graders were sleeping when they checked Farrell's paper so they took out a few points from his paper or what.

The one month that came between the announcement of qualifiers for the national stage and the actual national stage, which would be held on January 23, 2016, I spent studying and preparing for the national stage. I tried old PMO problems and USAJMO problems to prepare for the national stage. During that time, for example, I once went to the province and had no internet access for several days, so I asked a friend, Sophia Dominique Dizon, to text me some problems to think about during the long car trip. I solved problems with Sean on Facebook, and we would talk about olympiad problems from time to time.

The days leading up to the day of the national finals were filled with anticipation. Anticipation and fear. I would not comment so much as to my emotions on those days, as they are well-documented in other places.

We corresponded with the PMO committee over email. They asked for our pictures to be placed in the PMO booklet, and they sent us the letter informing our parents we were a part of the national stage. Then, January 23 came.

4 National Stage

The national stage was held on January 23, 2016, at the AMV Building, University of Santo Tomas, Manila. It was a day-long event, a four-and-a-half-hour long written phase in the morning, a lunch break, a two-hour long oral phase, then dinner and awarding ceremonies in the evening.

There were 20 national finalists in the PMO 2016. Here they are, along with what I can recall about them from memory:

- Clyde Welsey Ang, from Chiang Kai Shek College. Won a silver medal in the IMO 2015. He also won first place in the NCR area stage.
- Luke Matthews Bernardo, from Philadelphia High School. Won second in the NCR area stage.
- Elijamin Wolfgang Claveria, from Philippine Science High School – Main.
- Kyle Patrick Dulay, from Philippine Science High School – Main. Won an honorable mention in the IMO 2015 and second in the NCR area stage.
- Christian Philip Gelera, from Philippine Science High School – Main.
- Xavier Jefferson Ray Go, from Zamboanga Chong Hua High School. Won first in the Mindanao area stage.
- Andres Rico Gonzales III, from Colegio de San Juan de Letran. Being grade 8, he is the least senior of the finalists.

- Matthew Angelo Isidro, from Saint Jude Catholic High School.
- Andrea Jessica Jaba, from Saint Jude Catholic High School.
- Sedrick Scott Keh, from Xavier School.
- Dion Stephan Ong, from Ateneo de Manila High School. Also grade 8, making him least senior of the finalists.
- Tiffany Mae Ong, from Immaculate Concepcion Academy – Greenhills.
- Albert John Patupat, from Holy Rosary College. Won bronze in the IMO 2016, and first in the Luzon area stage.
- Shaquille Wyan Que, from Grace Christian College.
- Carl Joshua Quines (me), from Valenzuela City School of Mathematics and Science. Won third place in the NCR area stage.
- Errol John Suarez, from Aquinas University of Legaspi Science High School. Won third place in the Luzon area stage.
- Vince Jan Torres, from Santa Rosa Science and Technology High School. Won second place in the Luzon area stage.
- Sean Anderson Ty, from Zamboanga Chong Hua High School. Won first in the Mindanao area stage.
- Isabel Jocyn Villanueva, from PAREF Woodrose.
- Farrel Eldrian Wu, from MGC New Life Christian Academy. Being grade 11, he is the most senior competitor. Won bronze in the IMO 2016.

Of the 20 contestants, three contestants were from the Luzon area, two contestants were from the Mindanao area, and 15 contestants were from the NCR area. Sadly, no contestants were from the Visayas area. As is common with the PMO national stage, most of the contestants were from NCR, and most of the contestants are male, with only three being female.

Four out of the six members of the Philippine IMO 2015 team qualified, the remaining two (Raymond Joseph Fadri and Adrian Reginald Sy) now in college. Nine out of the 20 qualifiers for the PMO 2015 returned to the national stage this year. I went against some really good students, so of course I was nervous. But I was also excited.

4.1 Registration

I woke up early on that day to meet Mr. Logronio at our school. Mr. Logronio accompanied me to UST. We ate breakfast and went to the fourth floor of the AMV building for the registration.

We arrive at the fourth floor 30 minutes earlier than the scheduled registration at 7:30 AM. Some other competitors are already there. As always, they seemed to know each other, as they were busy talking to each other about lots of things. They all sat in the benches by the left side of the floor, while I instead chose to seat at a table in the back side of the floor, far from social interaction.

Sitting away from the crowd did not isolate me from social interaction, however. Errol Suarez and his mother sat at the same table as I and my trainer did. Thus, we were forced to socialize. Apparently he is also friends with some of the other contestants, unlike me, who was only friends with Sean.

I then realized that I had no idea what Sean looked like. His Facebook picture did not show his face, nor did his AoPS picture. I had never seen him before and I only knew him by name. He, on the other hand, would at least have an idea what I looked like from my Facebook picture.

I did enjoy talking with Errol. I learned he is from the Luzon area, and that they had to take a bus and travel quite a bit to get to UST. They were staying at a nearby hotel. I remember him telling his mother not to force him to take pictures with friends, and I agreed that I didn't like pictures as well.

The time for registration came, 7:30 AM, and there was still no registration table. The PMO staff seemed to be there arranging things, but nothing seemed like a place to register. There was also no sign that things were about to begin.

The minutes passed. Eventually there was no registration table, and the contestants were just asked to go to the examination room. I said goodbye to my trainer, as we were about to take a four-and-a-half hour examination.

The contestants were seated at the examination room. Again, I sat near Kyle, right behind him, in fact. I bought to my table my tumbler, a water bottle, a pen, a pencil, an eraser, a compass and a ruler. This was the first time I would use a compass and ruler in several months.

We were given a last-minute chance to go to the bathroom, and I do so. There was a bit of a mishap regarding the air conditioner, which was dripping, so they had to move the tables of some of the contestants.

We were seated in our respective seats. I then saw where Sean was seated, and then we exchanged glances and waved to one another after taking our seats. Finally I saw what Sean looked like.

They handed out the papers for the written phase of the PMO national stage, and we began at 8:00 AM. We would end at 12:30 PM, four-and-a-half hours after the beginning of the examination.

4.2 Written phase

It is anticipated that at the time of the publication of this document, the questions for the written phase have already been published by the PMO somewhere publicly. For the sake of ease of reference, I republish these here.

1. The operations below can be applied on any expression of the form $ax^2 + bx + c$.

(I) If $c \neq 0$, replace a by $4a - \frac{3}{c}$ and c by $\frac{c}{4}$.

(II) If $a \neq 0$, replace a by $-\frac{a}{2}$ and c by $-2c + \frac{3}{a}$.

(III) _{t} Replace x by $x - t$, where t is an integer. (Different values of t can be used.)

Is it possible to transform $x^2 - x - 6$ into each of the following by applying some sequence of the above operations?

(a) $5x^2 + 5x - 1$

(b) $x^2 + 6x + 2$



Figure 1: The written phase. I am seated at the front.

2. Prove that the arithmetic sequence $5, 11, 17, 23, 29, \dots$ contains infinitely many primes.
3. Let n be any positive integer. Prove that

$$\sum_{i=1}^n \frac{1}{(i^2 + i)^{3/4}} > 2 - \frac{2}{\sqrt{n+1}}$$

4. Two players, A (first player) and B , take alternate turns in playing a game using 2016 chips as follows: *the player whose turn it is, must remove s chips from the remaining pile of chips, where $s \in \{2, 4, 5\}$.* No one can skip a turn. The player who at some point is unable to make a move (cannot remove chips from the pile) loses the game. Who among the two players can force a win on this game?
5. Pentagon $ABCDE$ is inscribed in a circle. Its diagonals AC and BD intersect at F . The bisectors of $\angle BAC$ and $\angle CDB$ intersect at G . Let AG intersect BD at H , let DG intersect AC at I , and let EG intersect AD at J . If $FHGI$ is cyclic and

$$JA \cdot FC \cdot GH = JD \cdot FB \cdot GI,$$

prove that G , F and E are collinear.

I took a few minutes to stretch and I read the paper a bit before trying to solve any of the questions. The format of the paper was two algebra questions, one geometry, one combinatorics and one number theory, as is always the format of the PMO national stage. Which made me wonder, why don't they have two geometry questions? Or two combinatorics questions, or two number theory questions?

4.2.1 Problem 1

Problem 1 was an algebra question involving a quadratic. Upon seeing the problem, I try applying the operations on the given polynomial to get a feel for the problem. Basically operations I and II produce a lot of fractions, and I imagine that both (a) and (b) are impossible.⁴ I also observed that operations I and II leave b constant, while operation III changes b to $b - 2at$.

I think about it – what do I know quadratic polynomials? I tried thinking about the factored form, but that wouldn't help, then I tried looking at the roots. I then noticed that operation I and II seemed to change the roots in a pretty much unpredictable manner. But in finding the roots for the polynomials, I had to use the quadratic formula, which reminded me of the discriminant. And indeed, the discriminant of the polynomial was the key to this problem.

I looked at the discriminant of $ax^2 + bx + c$ after any of the operations. The original discriminant is $b^2 - 4ac$, and after applying operation I the discriminant becomes $b^2 - 4ac + 3$. After applying operation II, the discriminant becomes $b^2 - 4ac + 6$. Applying operation III leaves the discriminant unchanged. It clicked – the discriminant was monovariant nondecreasing, and it was invariant modulo 3.

I looked at the discriminant of the given polynomial, which was 25. The discriminant of (a) was 45, and 25 and 45 had different residues modulo 3 so that was impossible. The discriminant of (b) was 28, so looking at the discriminant alone isn't needed, but the key for (b) is noticing the coefficient b , which can only be changed through operation III to $b - 2at$. But for the given polynomials, $b - 2at$ can never equal 6, so (b) is impossible. I wrote those key observations on scratch paper and moved on to problem 2.

4.2.2 Problem 2

Problem 2 was the number theory question, proving the infinitude of primes for a certain arithmetic sequence. “Infinitude” and “primes” reminded me of Euclid's famous proof of the infinitude of primes, and the same underlying concept was under my proof of problem 2 – assume a finite number of primes, then construct a new one.

I noted that the terms of the sequence were -1 modulo 6. Assume there are a finite number of primes. If there are an even number, we multiply them all together and subtract 2. If there are an odd number, we multiply them all together and add 6. This produces a new prime number not in the original sequence. I write “Euclid” on my scratch paper and move to problem 3.

At this point, the proctors gave everyone there lunch allowance. Which I commend the PMO committee for doing.

4.2.3 Problem 3

Problem 3 was another algebra problem, proving a certain inequality. The inequality was anything but elegant and beautiful. It was easily the ugliest problem in the paper. I decided that the proof of this would be an induction, as we were going to prove something for all positive integers.

⁴It's more common to prove that something is impossible rather than possible in math competitions, but it's important never to decide on something too early.

quadrilaterals and cyclic hexagons. The given condition, $JA \cdot FC \cdot GH = JD \cdot FB \cdot GI$ was also quite uncommon, and it was difficult to turn that into anything.

Initially, I tried to see if I could prove collinearity using power of a point, since the given condition was about lengths, and power of a point deals with lengths. The setup was replete with points ripe for taking the powers of with a cyclic pentagon and a cyclic quadrilateral, so I listed a bunch of equations, and did a lot of division and manipulation, but I couldn't get anywhere.

I then went back to the fact that AG and DG are angle bisectors. I did a bit of angle chasing, as circles are fun for doing angle chasing in. I ended up proving $\angle AHD$ and $\angle AID$ were right, and that point G lies on the circle. I didn't immediately see how that would be useful for the problem.

At this point, I wrote my solutions for problems 1, 2 and 4. Problem 1 ended up pretty much like an essay, that managed to fill a whole page even with my small writing. Problem 2 was also a number theory essay, and I also filled a whole page even with small writing. Problem 4 was a combinatorics problem that asked for an essay by its very nature, so I happily wrote an essay for problem 4.

I reread my solutions for problems 1, 2 and 4. My solution for problem 4 was a bit messy and hard to follow but it was complete and correct and I believe it contained no errors. All the solutions at that time were far from neat – they all contained a few erasures, not to mention my penmanship was terrible.

4.2.6 Problem 3, again

I went back to problem 3, again. I still messed up my induction step, and only very narrowly managed to prove the inequality. I ended up manipulating the induction step to the point to proving a certain function was always less than or equal to $\frac{1}{4}$, and I said that that function was monovariant decreasing, so the induction was complete.

As I wrote my solution for problem 3, it did not end up like an essay like problems 1, 2 and 4 did. It ended up as a list of equations separated by occasional words, which was a relief to write, a stark contrast to my solutions to problems 1, 2 and 4.

I finish my writeup for problem 3 and try problem 5 again.

4.2.7 Problem 5, again

After highlighting the relevant segments from $JA \cdot FC \cdot GH = JD \cdot FB \cdot GI$ in my diagram, I noticed that it was very nearly Ceva's theorem. Then I decided that my method of proving the collinearity of J, F and G was using the converse of Ceva's theorem, which was correct because the condition for Ceva's was an 'if and only if'.

Proving the collinearity of J, F and G was the same as proving that Cevians AI, DH and GJ coincided at point F , which was the same as proving $JA \cdot DI \cdot GH = JD \cdot GI \cdot AH$. I looked at the given equation and realized that it already had four out of the six segments required in my proof, which was a good sign that I was at the right path.

I used power of a point on F , to show that $FA \cdot FC = FB \cdot FD$, so the equation became $JA \cdot FD \cdot GH = JD \cdot FA \cdot GI$. I had to turn FA and FD into DI and AH somehow, and power of a point using the cyclic quadrilateral $FHGI$ came to mind. I had $AH \cdot AG = AF \cdot AI$ and $DF \cdot DH = DI \cdot DG$. The equation then became $JA \cdot DI \cdot DG \cdot GH \cdot AI = JD \cdot AH \cdot AG \cdot GI \cdot DH$.

I was down to the last steps. If I could prove that $DG \cdot AI = AG \cdot DH$, my proof would be complete. At first, I looked at the absurdity of proving that and thought

of other ways to get from that equation to what I needed. But then I realized I had everything I needed: from my previous angle chase, I knew $\angle AID = \angle AHD = 90^\circ$. Thus, AI was perpendicular to DG and DH was perpendicular to AG . Then, the area of triangle ADG could be expressed as $\frac{AI \cdot DG}{2} = \frac{AG \cdot DH}{2}$, so $AI \cdot DG = AG \cdot DH$. With this, my proof for problem 5 was complete.

I took a bathroom break before writing up my solution for problem 5.

4.2.8 Post-solving

At that point I had already taken two bathroom breaks, which is clearly suspicious from any viewpoint. I then decided not to go to the bathroom until the end of the written phase. Which was clearly very difficult as both my tumbler and water bottle were empty, which meant I drank about 900 milliliters of water over the past three hours, and I knew that water had to go out of my body in one way or another.

I wrote up my solution for problem 5, which also ended up, much to my reluctance, as an essay. So I had an algebra essay, a combinatorics essay, a number theory essay and a geometry essay. I felt much sympathy for the graders.

I then reread my solutions for the problems and correct a few details. I ensured there were no missing steps, and there were missing steps in problems 2 and 4, which I added in using a footnote.

At that point I had about 45 minutes before the exam would end, so I spent that time doodling. I organized my things neatly, collating all my papers together. Kyle seemed to be finished early, as his things were neatly organized and he was simply staring at his solution. The proctors reminded us that if we are finished, we may leave early. I have elected not to leave early as that would be bad manners.

Minutes passed and more people seemed to be finished, however, no one chose to leave early. The proctors distributed clips to everyone to clip our solution pages together. I overheard the organizers talking to each other, about the binder clips, and it seemed that they were going to use envelopes originally.

By then, I really needed to go to the bathroom. But I decided to hold it in as going in the bathroom thrice would be too impolite.

The proctors reminded us to write our code and not our name in the upper right corner of each page of our solution, to label and encircle the problem number at the upper right corner of each page of our solution, and to label the pages of each solution per problem, page 1 of 3, page 2 of 3, and so on.

Someone, I didn't notice who, asked about the page labelling, and the proctor asked how many pages his solution for problem 1 had, and he said it had six pages. I thought about what a six-page solution of problem 1 would have. It would most likely start with a definition of a discriminant and an introduction to the use of it in quadratic equations.

4.3 Lunch

Finally, the time was finished. We all submitted our papers and I organized my things. And of course, the inevitable happened, everyone immediately formed groups to discuss the questions. At least at that time I managed to form a group with Sean and Xavier Jefferson Go, so I had someone to talk to.

We were both able to solve problem 1. We also solved problem 2 in the same way, in the same vein as Euclid. Then I noticed that my solution for problem 3 has a

fatal error – the function that I tried to prove was less than or equal to $\frac{1}{4}$ was in fact monovariant increasing, not monovariant decreasing, shattering the validity of my induction entirely. I said I didn't solve problem 3, and he asked me why not, since it was easily solvable by induction. I said I did an induction, but I messed up my inductive step and called the base case trivial.

We both managed to solve problem 4, but we had different answers. I said B would force a win, while both Sean and Xavier told me they answered A could force a win. I then immediately doubted the validity of my solution, as I am often wrong when it comes to little details. They doubted the validity of their solutions as well.

Sean told me he tried to complex-bash problem 5, but to no avail. I immediately felt sorry for him, as problem 5 wasn't suitable for complex-bashing at all. I told him I solved it synthetically through the converse of Ceva's theorem. It turns out that he didn't think of using Ceva's theorem.

At that point the proctor was trying to get all the students out of the room as they needed to prepare the room for the oral round. As previously stated, it is envitable that students will talk about the problems after a math competition. The proctors did manage to get us all out of the room. They told us to return at 1:30 PM after lunch for the oral phase.

I met with my trainer, Mr. Logronio. He was talking to the mother and trainer of Errol. Errol also met with his trainer and mother. I told Mr. Logronio that although I finished an hour early, I chose not to leave early. He asked me why, and I told him that it's impolite to the other contestants. He laughed, and he also told me that leaving early would be a gesture of pride.

We went down from the fourth floor, and the hive mind of the PMO finalists took us to a fast-food chain. Mr. Logronio initially lined up at the wrong line and I corrected him. I found a seat upstairs and waited for him. The other contestants ate their meals together, but I ate alone with Mr. Logronio. Shortly after Mr. Logronio went up, bringing our lunch, Mary Ann Balmes, a mathematics teacher also from our school, came to our table.

It turned out that Ms. Balmes came to us from attending a class. Mr. Logronio offered her lunch, but she declined. We talked about the problems and about our families during lunch. After a few minutes, we went back to the fourth floor for the oral phase.

4.4 Oral phase

I arrived at the hall for the oral phase thirty minutes early, so I had some time to spend acclimating to the environment. I texted my mom and asked her whether or not she will be able to come to watch the oral phase, and she texted that she was on her way.

Some of the PMO staff began handing out the PMO 2016 booklets to the audience. I managed to acquire a copy. It had the usual messages from very important people at the front, a list of the people of the PMO team, which made me realize how much work goes into preparing a competition of this size, the winners of the area stage, the prizes, a list of the finalists of the PMO, with pictures, highlights from the PMO 2015, the qualifying stage, answers to the qualifying stage, the area stage, answers to the area stage, and advertisements from the sponsors.

I spent a few minutes looking at the pictures of the contestants in an attempt to



Figure 3: The oral phase. I am seated at the back.



Figure 4: The trophies and medals. They look really nice.



Figure 5: The three judges, whom enjoyed the oral phase while subjecting us to the equivalent of mathematical torture.

associate their names with their faces. I looked at the answers to the qualifying and area stages, and noted that the answers to the qualifying stage were wrong.

Sean and Xavier finished eating their lunch and go to the hall. I saw them, and we sat together. We talked about lots of different things, like math, my family, and of course, the written phase. Kyle sat behind us and he was working on a problem.

While I was talking with Sean and Xavier, the staff played the timer for the oral phase. Each second was marked with a ticking sound, which was effective in providing a sense of time pressure. Sean said that if that sound was going to play during the oral phase he might not be able to answer anything. I wholeheartedly agreed.

A few minutes passed, and we were called to the front, along with our trainer, to be awarded our certificates of participation to the national stage. Then the oral phase began.

We were seated in a really large table for one person to solve math problems in. I only occupied about one-sixth of the table. We were provided with scratch paper and answer sheets.

The oral phase consisted of thirty questions. The easy round would have fifteen fifteen-second two-point questions, the average round would have ten thirty-second three-point questions, the difficult round would have five sixty-second six-point questions.

The first few questions were read and indeed the easy round was very difficult to solve within the time limit of fifteen seconds, especially since the ticking sound was played in the background. The questions themselves I feel do not deserve much remark, except that they were relatively straightforward if you had five minutes to solve instead of fifteen seconds. As PMO custom, the first fourteen questions of the easy round are problems, while the last question is a history question. This year's question was for the meaning of the acronym GIMPS, in connection with their recent discovery of the current largest prime. Only Farrell and I got that question correctly so I felt happy, if only for a bit.

The easy round ended and the clear leaders were Kyle, Clyde Welsey Ang, Albert

John Patupat and Farrell. The average round questions were also pretty much unremarkable. The difficult round questions were painful in difficulty and only a handful of people managed to solve a single question in the difficult round. We were told later that two of the judges for the oral phase thought of some of the difficult round questions.

The oral phase ended at about 3:00 PM. First in the oral phase was Farrell, followed by Kyle, Albert and Clyde, though I can't remember the exact order. I was ranked eight among the 20 contestants in the oral phase, tied with something like three other people.

I have no intentions in discussing the oral phase's questions in more detail. At the time of writing, they are currently not at the PMO website, though it is anticipated that the questions will be posted in the near future.

4.5 Pre-awarding

With the oral phase ending two hours earlier than its scheduled end at 5:00 PM, we had three hours to burn while waiting for the dinner and awarding at 6:00 PM. My mother came to the hall immediately after the oral phase ended, so I met her, and she met Mr. Logronio and Ms. Balmes.

Sean invited me to his hotel room to spend the three hours with him while waiting for the awarding. I declined, saying that my mother is with me. I regret that decision a lot, even until now, I think about what could have happened if I said yes.

We sat at the table at the back of the floor, while my mom and Ms. Balmes talked about things. Mr. Logronio already left us as he had a class to attend. We waited for Ms. Soriano to come, who was also coming from a class. After a few minutes of talking and staying under the heat of the sun, we decided to get a bite to eat.

We ate at a small shop downstairs, and I ate a banana split. Dion Stephan Ong, along with his companions, also happened to eat at the same place we did. We briefly exchanged glances and waved at each other.

Eventually Ms. Soriano came to the shop and she talked with my mom. I showed Ms. Soriano a copy of the national stage. After a few more minutes of waiting and eating and talking, we went up for the dinner and awarding ceremonies.

4.6 Awarding

After going up to the fourth floor, I changed to a nice long-sleeved polo. It was, in fact, my father's, and I had already grown to his size. The staff gave us a copy of the awarding booklet, as well as showed us our table assignments. It was a good thing that I was seated next to the other contestants, as it would be a great opportunity to get to know the people whom I always see and read about. Although I was only allowed to bring two people with me to the dinner, Ms. Soriano was allowed in as the representative of our principal.

I arrived at the dinner 30 minutes early. Some of the contestants were early as well. I waited for the awarding to begin as the room filled up with people. Sean came and he sat to my left, Xavier sat on Sean's left. Kyle and Albert sat on our table, as well as Clyde, Luke Matthews Bernardo and Elijamin Wolfgang Claveria.

I enjoyed talking with Sean and Xavier about lots and lots of things. We were then handed out a 2016 Math Olympiad Summer Camp Participation Information and Questionnaire, which invited us to the MOSC 2016, a six-week training and



Figure 6: The twenty national finalists. Top row, from left: Shaquille, Xavier, Clyde, Vince, Farrell, Albert, Luke, Christian, Matthew, Elijamin. Bottom row, from left: Kyle, Sedrick, Sean, CJ (me), Dion, Andrea, Isabel, Tiffany, Andres, Errol. I think – I suck with names.



Figure 7: Me (center), standing between four people whom are apparently very important but whom I forgot the names of. Sorry.

selection venue for the IMO. One of the questions in the questionnaire was to assess one's mathematical skill, and I perhaps overestimated my skill by a little bit. Other questions included whether or not one could take the APMO or attend the MOSC, and I answered undecided for both.

Eventually the ceremonies began with an invocation and national anthem. Then the usual speeches were made, and I tried to listen to at least the keynote speech. Apparently Sean was working on a problem while the speeches were being delivered, a number theory problem on divisibility. Soon enough pretty much everyone within a one-meter radius of me learned about the problem and worked on it. Most of the other contestants have split themselves to their own groups.

Then the dinner was held, and we lined up at the buffet. Sean asked what one of the dishes was, and the person serving the dish had to ask the person next to him what it was. It was spare ribs.

The dinner was satisfying. Xavier tried to get the attention of the waiter pouring out water, and failed to do so thrice. It was good that I managed to get the waiter's attention the fourth time and they poured water for Xavier.

I then tried remembering the names of everyone at our table. I did manage to remember all their names, and I think that I'd recognize them if I see them again. Not so sure if they'd recognize me, however...

Several intermission numbers were given. It turned out that not all mathematically inclined people are talentless as the voices of one faculty member and one student turned out to be pretty good. The faculty member sang *Without You*, and I couldn't recognize what the student sang but he definitely messed up a chord somewhere.

Then medals are handed out to the area stage winners and the finalists, and lots and lots of pictures are taken. Meanwhile, Sean still hasn't solved the number theory problem from earlier. Albert seems to have made some progress. Plaques of appreciation are handed out to the sponsors of the PMO.

Finally, the moment everyone waited for arrived, where the winners of the national finals were announced. First was Farrell, second was Kyle and third was Albert. I definitely predicted the top three correctly, and I said to Sean that I told him so.

Then there were two more speeches, but they were slightly more important speeches as they concerned the aftermath of the PMO, the Philippine team for the IMO and the MOSC. So of course everyone listened.

The closing speech was given and then the night was already over. I told Sean that I wished that that night would last longer, because I had a lot of fun being with him and the other contestants, of talking to people whom I can relate to mathematically. He agreed. I said goodbye, and I told him that I don't know when I'll see him again, because I wasn't sure about whether or not I can attend the MOSC.

5 Closing

The PMO was very enjoyable. It is my hope that I am able to perform well in the MOSC in order to qualify for the Philippine team, but we shall see. In any case, I do sincerely hope that I can make it again to the national stage of the PMO 2017.

A lot of work has clearly been placed behind the whole event, and everything went smoothly and without major error. The whole event was very memorable. I would like to thank thusly the organizers of the PMO for making that possible.



Figure 8: The two medals I won. They're plexiglass and they're really heavy.

Thanks as well to the problem setters for the papers, because if they made the questions for the area stage too difficult I might not have made it to the national stage. That aside, I would like to thank them for creating beautiful, elegant and original problems that inspire creative thought.

I would never have been mathematically inclined if it was not for the support of the Valenzuela City School of Mathematics and Science mathematics department. All of the teachers of the math department at our school are truly special and passionate about mathematics, and I would never have become as good as I am now if it wasn't for their encouragement and teaching.

Thanks to the competitors of the PMO for providing good company during the conduct of the competition, especially to Sean. It was wonderful getting to know them and talking to them, and it was happy to be with them throughout the PMO.

5.1 Post-script

It is February 26, 2016, a month after originally writing this. (I originally wrote this on January 29, 2016, a week after the PMO national stage.) I have made a few minor edits and added pictures. I have refused the temptation to make major edits to my work, as I would like this to be a reflection of what I felt after the competition. Lots of things have changed – the people who were once strangers are now my friends. So that's nice. The world of mathematics here in the Philippines is a small one, and you run into the same people over and over again, so it pays to get to know them. And it's nice to have friends you can relate to.