VCSMS PRIME

Session 2: Trigonometry
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Circular functions

1. (11QI8) Find the sum $\cos 1^\circ + \cos 3^\circ + \cos 5^\circ + \cdots + \cos 177^\circ + \cos 179^\circ$.
2. (13QI15) Find the value of $\sin\theta$ if the terminal side of θ lies on $5y - 3x = 0$ and θ is in the first quadrant.
3. (11AI14) The line from the origin to the point $(1, \tan 75^\circ)$ intersects the unit circle at P. Find the slope of the tangent line to the circle at P.
4. (11AI11) Find the sum of the coefficients of the polynomial $\cos(2\cos^{-1}(1-x^2))$.

Identities

1. (11QII5) Find the value of $\cos 15^\circ$.
2. (14QII6) Evaluate $\log_2 \sin(\pi/8) + \log_2 \cos(15\pi/8)$.
3. (16NE9) If $\tan x + \tan y = 5$ and $\tan(x+y) = 10$, find $\cot^2 x + \cot^2 y$.
4. (15AI4) Find the numerical value of $(1 - \cot 37^\circ)(1 - \cot 8^\circ)$.
5. (16NA1) Find the value of $\cot(\cot^{-1} 2 + \cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5)$.
6. (16AI6) Evaluate $\displaystyle\prod_{\theta=1}^{89}(\tan\theta^\circ \cos 1^\circ + \sin 1^\circ)$.
7. (13AI14) Given that $\tan\alpha + \cot\alpha = 4$, find $\sqrt{\sec^2\alpha + \csc^2\alpha - \frac{1}{2}\sec\alpha\csc\alpha}$.

Equations

1. (13QI11) If $2\sin(3x) = a\cos(3x + c)$, find all values of ac.
2. (13QI10) How many solutions has $\sin 2\theta - \cos 2\theta = \sqrt{6}/2$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?
3. (10NA9) If $0 < \theta < \pi/2$ and $1 + \sin\theta = 2\cos\theta$, determine the numerical value of $\sin\theta$.
4. (13NE13) Find the solution set of the equation $\dfrac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = 2$.
5. (10ND4) Find the only value of x in $(-\pi/2, 0)$ that satisfies $\dfrac{\sqrt{3}}{\sin x} + \dfrac{1}{\cos x} = 4$.
6. (16AI13) Find all real numbers a and b so that for all real numbers x,
$$2\cos^2\left(x + \frac{b}{2}\right) - 2\sin\left(ax - \frac{\pi}{2}\right)\cos\left(ax - \frac{\pi}{2}\right) = 1.$$
7. (14AI12) Suppose $\alpha, \beta \in (0, \pi/2)$. If $\tan\beta = \dfrac{\cot\alpha - 1}{\cot\alpha + 1}$, find $\alpha + \beta$.
8. (14ND3) Find all $0 \le \theta \le 2\pi$ satisfying $\sqrt{\dfrac{1}{2} + \dfrac{1}{2}\sqrt{\dfrac{1}{2} + \dfrac{1}{2}\sqrt{\dfrac{1}{2} + \dfrac{1}{2}\cos 8\theta}}} = \cos\theta$.

Triangle laws

1. (16NE5) In right triangle $ABC, \angle ACB = 90^\circ$ and $AC = BC = 1$. Point D is on AB such that $\angle DCB = 30^\circ$. Find the area of $\triangle ADC$.
2. (13NE11) In $\triangle ABC, \angle A = 60^\circ, \angle B = 45^\circ$, and $AC = \sqrt{2}$. Find the area of the triangle.
3. (10QIII5) Let M be the midpoint of side BC of triangle ABC. Suppose that $AB = 4, AM = 1$. Determine the smallest possible measure of $\angle BAC$.
4. (13AI9) Consider an acute triangle with angles α, β, γ opposite the sides a, b, c respectively. If $\sin\alpha = \dfrac{3}{5}$ and $\cos\beta = \dfrac{5}{13}$, evaluate $\dfrac{a^2 + b^2 - c^2}{ab}$.
5. (15AII3) Points A, M, N and B are collinear, in that order, and $AM = 4, MN = 2, NB = 3$. If point C is not collinear with these four points, and $AC = 6$, prove that CN bisects $\angle BCM$.
6. (11AII2) Denote by a, b, c the sides of a triangle opposite angles α, β, γ, respectively. If $\alpha = 60^\circ$, prove that $a^2 = \dfrac{a^3 + b^3 + c^3}{a + b + c}$.