VCSMS PRIME

Program for Inducing Mathematical Excellence

October 27, 2017

Session 12: Metasolving

Best practice

Best practice

1. Reread the question.

Best practice

1. Reread the question.
2. Work cleanly.

Best practice

1. Reread the question.
2. Work cleanly.
3. Be aware of your time.

Best practice

1. Reread the question.
2. Work cleanly.
3. Be aware of your time.
4. Check your work.

Best practice

1. Reread the question.
2. Work cleanly.
3. Be aware of your time.
4. Check your work.
5. Learn how to guess.

How to read

How to read

- Most common source of mistakes is misreading.

How to read

- Most common source of mistakes is misreading.
- Details: integer vs. positive integer, complex vs. real.

How to read

- Most common source of mistakes is misreading.
- Details: integer vs. positive integer, complex vs. real.
- Can't make progress? Reread.

How to read

- Most common source of mistakes is misreading.
- Details: integer vs. positive integer, complex vs. real.
- Can't make progress? Reread.
- Remember all details: most likely all will be used.

How to read

- Most common source of mistakes is misreading.
- Details: integer vs. positive integer, complex vs. real.
- Can't make progress? Reread.
- Remember all details: most likely all will be used.
- Reread after answering. Proper format? Correct units?

How to write

How to write

- Next most common source of mistakes is misreading...

How to write

- Next most common source of mistakes is misreading...
- ...your own handwriting.

How to write

- Next most common source of mistakes is misreading. . .
- . . . your own handwriting.
- Write neatly and legibly.

How to write

- Next most common source of mistakes is misreading...
- ...your own handwriting.
- Write neatly and legibly.
- And unambiguously: ℓ vs. l, 1 vs. 7, x vs. y.

Time management

Time management

- Trade-off: how much time solving vs. checking your work?

Time management

- Trade-off: how much time solving vs. checking your work?
- Always know how much time is left.

Time management

- Trade-off: how much time solving vs. checking your work?
- Always know how much time is left.
- Wear a watch.

Checking

Checking

- Correcting a mistake is faster than solving.

Checking

- Correcting a mistake is faster than solving.
- Fast checking methods: plugging in, different method, examples.

Checking

- Correcting a mistake is faster than solving.
- Fast checking methods: plugging in, different method, examples.
- Mark unsure problems.

Checking

- Correcting a mistake is faster than solving.
- Fast checking methods: plugging in, different method, examples.
- Mark unsure problems.
- Do not repeat solutions.

Meta on checking

Meta on checking

- Finish the exam early: check.

Meta on checking

- Finish the exam early: check.
- When you have a few minutes left: check.

Meta on checking

- Finish the exam early: check.
- When you have a few minutes left: check.
- Rarely catch your own mistakes? Don't check.

Meta on checking

- Finish the exam early: check.
- When you have a few minutes left: check.
- Rarely catch your own mistakes? Don't check.
- Usually more efficient to check than solve.

Meta on checking

- Finish the exam early: check.
- When you have a few minutes left: check.
- Rarely catch your own mistakes? Don't check.
- Usually more efficient to check than solve.
- Error-prone? More checking time.

Guessing

The sum of four two-digit numbers is 221, none of the eight digits are 0, and no two digits are the same. Which of these are not included among the eight digits?

(a) 2 (b) 4 (c) 6 (d) 8

Guessing

A digital watch displays hours and minutes with AM and PM. What is the largest possible sum of digits in the display?

(a) 17 (b) 19 (c) 21 (d) 23

Meta-guessing

Meta-guessing

(a) $(-2, 1)$ (b) $(-1, 2)$ (c) $(2, -1)$ (d) $(1, -2)$ (e) $(4, 4)$

Meta-guessing

(a) $(-2, 1)$ (b) $(-1, 2)$ (c) $(2, -1)$ (d) $(1, -2)$ (e) $(4, 4)$

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{5}{6}$ (e) $\frac{9}{4}$

Meta-guessing

(a) $(-2, 1)$ (b) $(-1, 2)$ (c) $(2, -1)$ (d) $(1, -2)$ (e) $(4, 4)$

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{5}{6}$ (e) $\frac{9}{4}$

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

Meta-guessing

(a) $(-2, 1)$ (b) $(-1, 2)$ (c) $(2, -1)$ (d) $(1, -2)$ (e) $(4, 4)$

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{5}{6}$ (e) $\frac{9}{4}$

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

(a) 2 (b) $\frac{1}{2}\pi$ (c) π (d) 2π (e) 4π

Abuse

Two non-zero real numbers a and b satisfy $ab = a - b$. Find a possible value of $a/b + b/a - ab$.

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

Abuse

Let a, b, c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Find $a^2 - b^2 + c^2$.

(a) 0 (b) 1 (c) 4 (d) 7 (e) 8

Abuse

In triangle ABC, BD is the angle bisector of $\angle ABC$, and $AB = BD$. Moreover, E is a point on AB such that $AE = AD$. If $\angle ACB = 36°$, find $\angle BDE$.

(a) $24°$ (b) $18°$ (c) $15°$ (d) $12°$

Elimination

How many ordered triples (a, b, c) of non-negative integers satisfy $a + b + c = 6$?

(a) 22 (b) 25 (c) 27 (d) 28 (e) 29

Elimination

Let n be a five-digit number. Suppose that when n is divided by 100, its quotient is q and the remainder is r. For how many values of n is $q + r$ divisible by 11?

(a) 8180 (b) 8181 (c) 8182 (d) 9000 (e) 9090

Elimination

What non-zero value of x satisfies $(7x)^{14} = (14x)^7$?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) 1 (d) 7 (e) 14

Problem solving

Problem solving

1. What is problem-solving, really?

Problem solving

1. What is problem-solving, really?
2. Can we make ourselves better problem-solvers?

Problem solving

1. What is problem-solving, really?
2. Can we make ourselves better problem-solvers?
3. How do people solve problems anyway?

Two parts

Two parts

- Exploration and motivation.

Two parts

- Exploration and motivation.
- Explore: read and understand problem, draw diagrams, small cases, make tables, get hands dirty.

Two parts

- Exploration and motivation.
- Explore: read and understand problem, draw diagrams, small cases, make tables, get hands dirty.
- Motivation is the “magic”, “lightbulb moment”, “sudden realization”, “intuition”.

Intuition

Intuition

- Mostly intuition: “hard to describe”, “unknown”. Often cause of doubt: “is it legit”?

Intuition

- Mostly intuition: “hard to describe”, “unknown”. Often cause of doubt: “is it legit”?
- “It’s ust gut feeling, maybe even luck when you put it into context.”

Intuition

- Mostly intuition: “hard to describe”, “unknown”. Often cause of doubt: “is it legit”?
- “It’s ust gut feeling, maybe even luck when you put it into context.”
- “It’s the invisible guiding force in a mathematician’s attempts to solve problems.”

Intuition

- Mostly intuition: “hard to describe”, “unknown”. Often cause of doubt: “is it legit”?
- “It’s ust gut feeling, maybe even luck when you put it into context.”
- “It’s the invisible guiding force in a mathematician’s attempts to solve problems.”
- “It’s pattern recognition from previous problems you’ve solved.”

Motivation

Motivation

- *Intuition is recognition!*

Motivation

- *Intuition is recognition!*
- Simplifying the problem,

Motivation

- *Intuition is recognition!*
- Simplifying the problem,
- making things easier,

Motivation

- *Intuition is recognition!*
- Simplifying the problem,
- making things easier,
- noticing something.

Can we be better problem solvers?

Can we be better problem solvers?

- Answer: *yes*! Schoenfeld 1985.

Can we be better problem solvers?

- Answer: *yes*! Schoenfeld 1985.
- Exposure produces recognition. Example.

Can we be better problem solvers?

- Answer: *yes*! Schoenfeld 1985.
- Exposure produces recognition. Example.
- Not just practice, but also thinking about practice.