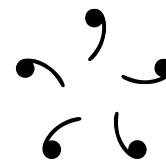# VCSMS PRIME

Program for Inducing Mathematical Excellence

Week 1 Homework

Due September 20, 2017

Homework

Each person is assigned two sets based on their problem-solving level. For the first set assigned to you, you only need to submit your answers. For the second set, you must submit *neat and orderly* solutions for each problem. Write your answers on short bond paper, write your name, and leave margins.

Problems refer to VCSMS PRIME 2016, on `http://cjquines.com/files/prime.pdf`, which is also posted in our Facebook group. For example, "**S9**: Circles 3–5, 7" means problems 3, 4, 5 and 7 in the Circles section of PRIME 2016 session 9.

Due on Week 1 due date: Wednesday, September 20.

Set A (13) **S1**: Domain and Range 5; Logarithms 1–2; Exponents 1–2; Value-finding 1.
   S7: Angles 1; Areas 1–3. **S8**: Surds 1–3.

Set B (15) **S1**: Domain and Range 6–7; Logarithms 3–6; Exponents 3–4; Floor, ceiling, fractional 1.
   S5: Equations 7. **S7**: Angles 2–3; Areas 4–5. **S8**: Surds 4.

Set C (16) **S1**: More logarithms: 1–2, 6–7; Floor, ceiling, fractional 2; Value-finding 2–3; Cauchy functional
   equation 1–3. **S5**: Equations 8. **S7**: Angles 4; Areas 6–8. **S8**: Surds 5.

Set D (14) **S1**: More logarithms: 3–5; Floor, ceiling, fractional 3; Other functional equations 1–5.
   S7: Angles 5; Areas 9–11. **S8**: Surds 6.

Additional problems

These problems are optional. They range from easy to very hard.

1. ABC is an isosceles triangle such that $AC = BC$. CBD is an isosceles triangle such that $CB = DB$. A and D are on the same side of line BC, and segments BD and AC intersect at a right angle. If $\angle A = 57°$, what is $\angle D$?

2. Quadrilateral $CFDE$ is inscribed in a circle with center O, which lies on segment CD. Lines CE and DF intersect at A and lines DE and CF intersect at B. If $\angle EAD = 40°$ and minor arc ED measures $40°$, find $m\angle DAB$.

3. Two equilateral triangles ABC and ADE are drawn, both with side length 4, such that segments DE and BC intersect and AD and BC are perpendicular. Find the area of the region common to both triangles.

4. Pentagon $ABCDE$ with side lengths $AB = BC = CD = 10, DE = 16, EA = 12$ is inscribed in a circle. If $\angle DEA = 90°$, find its area.

5. (Mathira 2017) In the figure to the right, the nine circles all have radius 1 and adjacent circles are either externally tangent or pass through each others' centers. Find the area of the shaded region.

6. (AII2) Let BH be the altitude from the vertex B to the side AC of an acute-angled triangle ABC. Let D and E be the midpoints of AB and AC, respectively, and F the reflection of H across the line segment ED. Prove that the line BF passes through the circumcenter of $\triangle ABC$.