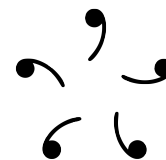


# VCSMS PRIME

Program for Inducing Mathematical Excellence

Week 2 Homework

Due September 27, 2017



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## Homework

Due on Wednesday, September 27. Reminder: you can always ask me or your classmates for help, it's okay!

**Set A** (13) **S4:** Ad hoc 2; Inclusion-Exclusion 1–2, 4; Permutations 1–2; Combinations 1; Balls and urns 1–2.  
**S6:** Random variable 1–2; Random selection 1–2.

**Set B** (13) **S4:** Ad hoc 3; Inclusion-Exclusion 3, 5; Permutations 3–4; Combinations 2; Balls and urns 3–4.  
**S6:** Random selection 3–5; Geometric probability 1; Existence combinatorics 1.

**Set C** (13) **S4:** Ad hoc 5; Permutations 5, 8; Combinations 3; Balls and urns 5–6.  
**S6:** Random variable 3; Random selection 6–8; Geometric probability 2–3; Existence combinatorics 2.

**Set D** (13) **S4:** Ad hoc 10; Permutations 6–7; Balls and urns 7.  
**S6:** Random variable 4; Random selection 9; Geometric probability 4; Existence combinatorics 3–8.

## Additional problems

Reminder: if you want more problems for practice I have some to give, just name a topic. This one includes some topics that haven't appeared in past PMOs but can appear in the future.

1. (AIME II 2004/4) How many positive integers less than 10,000 have at most two different digits?
2. (OMO Winter 2012/2) How many ways are there to arrange the letters AAAHH so that the sequence HA appears at least once?
3. (OMO Spring 2015/6) We delete the four corners of an  $8 \times 8$  chessboard. How many ways are there to place eight non-attacking rooks on the remaining squares?
4. (AIME I 2001/6) A fair die is rolled four times. What is the probability each of the final three rolls is at least as large as the roll preceding it?
5. (AIME I 2008/11) Consider sequences that consist entirely of *As* and *Bs* such that every run of consecutive *As* has even length, and every run of consecutive *Bs* has odd length. Examples are *AA*, *B*, and *AABAA*, while *BBAB* is not such a sequence. How many such sequences have length 14?
6. (SASMO 2015) Cheryl gives Albert and Bernard a list of 10 possible dates for her birthday: May 15, 16, 19; June 17, 18; July 14, 16; and August 14, 15, 17. Cheryl tells Albert the month of her birthday, and tells Bernard the day of her birthday. Given the following conversation, when is Cheryl's birthday?  
Albert: "I don't know when Cheryl's birthday is, but I know that you don't know either."  
Bernard: "At first I didn't know when Cheryl's birthday is, but now I know."  
Albert: "Now I also know when Cheryl's birthday is."
7. (AMC 12A 2007/25) Call a set of integers *spacy* if it contains no more than one out of any three consecutive integers. How many subsets of  $\{1, 2, 3, \dots, 12\}$ , including the empty set, are spacy?
8. (AIME 1986/13) How many different sequences of 15 coin tosses contain exactly two pairs of consecutive heads, three pairs of consecutive heads then tails, four pairs of consecutive tails then heads and five pairs of consecutive tails?