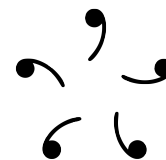


# VCSMS PRIME

Program for Inducing Mathematical Excellence

Week 3 Homework

Due October 4, 2017



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## Homework

Due on Wednesday, October 4. The additional problems this week are *required* based on your set.

**Set A** (13) **S2**: Circular functions 1–2; Identities 1–2; Triangle laws 1–2.

**S7**: Circles 1–3; Three-dimensional 1. **Additional problems**: 1–3.

**Set B** (13) **S2**: Circular functions 3–4; Identities 3; Equations 1–2; Triangle laws 3.

**S7**: Circles 4–5; Three-dimensional 2–3. **S9**: Ad hoc 1; Triangles 1. **Additional problems**: 4.

**Set C** (13) **S2**: Identities 4–5, 7; Equations 3–6; Triangle laws 4.

**S7**: Circles 6; Three-dimensional 4. **S9**: Ad hoc 2; Triangles 2. **Additional problems**: 5.

**Set D** (13) **S2**: Identities 6; Equations 7–8; Triangle laws 5–6.

**S7**: Three-dimensional 5. **S9**: Ad hoc 3–5; Triangles 3–5. **Additional problems**: 6.

## Additional problems

Again, the additional problems this week are *required*. Set A has 1–3, set B has 4, set C has 5 and set D has 6.

1. Vincent is solving a problem: “Two circles have radii 3 and 27, and the length of a common external tangent is 40. What is the distance of their centers?” However, he misread and thought it was “common *internal*” tangent, and answered the problem correctly assuming this. What is the difference between Vincent’s answer and the actual correct answer?
2. (AIME 1994/2) A circle with diameter  $PQ$  of length 10 is internally tangent at  $P$  to a circle of radius 20. Square  $ABCD$  is constructed with  $A$  and  $B$  on the larger circle,  $CD$  tangent to  $Q$  to the smaller circle, and the smaller circle outside  $ABCD$ . Find the length of  $AB$ .
3. (AIME 1991/2) Rectangle  $ABCD$  has  $AB = 4$  and  $CB = 3$ . Divide  $AB$  into 168 congruent segments with points  $A = P_0, P_1, \dots, P_{168} = B$ , and divide  $CB$  into 168 congruent segments with  $C = Q_0, Q_1, \dots, Q_{168} = B$ . For  $1 \leq k \leq 167$ , draw the segments  $P_k Q_k$ . Repeat this construction on the sides  $AD$  and  $CD$ , and then draw the diagonal  $AC$ . Find the sum of the lengths of the 335 parallel segments drawn.
4. (AHSME 1970) In trapezoid  $ABCD$ , we have  $AB \parallel CD$  and  $\angle B = 2\angle D$ . The length of  $AB$  can be represented as  $k$  times the length of  $AD$  plus  $\ell$  times the length of  $CD$ . What is  $k + \ell$ ?
5. (AIME 1998/6) Let  $ABCD$  be a parallelogram. Extend  $DA$  through  $A$  to a point  $P$ , and let  $PC$  meet  $AB$  at  $Q$  and  $DB$  at  $R$ . Given that  $PQ = 735$  and  $QR = 112$ , find  $RC$ .
6. A quadrilateral circumscribed about a circle has two adjacent right angles. The sides adjacent to one right angle have lengths 4 and 7. Find the radius of the inscribed circle.

## Additional reading

- Complex Numbers in Trigonometry, <https://aops.com/community/c6h609795>.
- Characterizations of Trapezoids, <http://forumgeom.fau.edu/FG2013volume13/FG201305.pdf>.