EASY: 2 points each

1 Consider an angle that intercept an arc whose length is $\frac{1}{400}$ of the circumference. Denote this angle by $1^{\triangle}$. If the circle has radius 2, find the area of the sector whose central angle is $30^{\triangle}$.

2 Daniel is standing a few feet away from a mirror hanging on the wall. He is 160 cm tall and is wearing a big hat that adds 30 cm. If his eye level is 150 cm from ground level, and he can see both the tip of his hat and his feet while standing straight, then what is the minimum possible height measurement of the mirror?

3 How many different isosceles triangles with perimeter 28 units can be formed where all three sides have whole number lengths?

4 What is the units digit of $3^{2007^{2008}}$?

5 Define $f\left(x+\frac{1}{x}\right)$ as $x^3-\frac{1}{x^3}$. Find $f(x)$.

6 Solve for x: $9^x+9^{-x}=\frac{10}{3}$.

7 Compute for the smallest positive number of the form p^3+7p^2 that is a perfect square.

8 The graphs of the functions $f(x)=ax+b$ and $g(x)=bx-a$ are perpendicular lines. If $\frac{g(1)}{f(0)}=5$ and $a>0$, what is the numerical value of a?

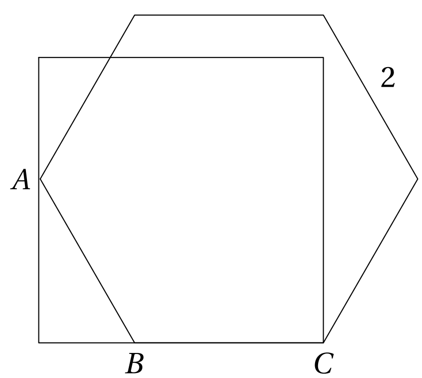
AVERAGE: 3 points each

1 Find the value of the term which does not involve x in the expansion of $\left(x^4-\frac{3}{x^3}\right)^7$.

2 The equation of a circle is given by $x^2+y^2-6x-6y+\frac{47}{4}=0$. If a line tangent to the circle passes through a point P, then the tangent segment is the line segment from P to the point of tangency. Find the length segment from $(11,-3)$ to the circle.

3 How many positive integers less than 10 000 have at most two different digits?

4 In the figure, as shown, a square is overlapping a regular hexagon with side length 2. Vertex A lies on one side of the square, $\overline{BC}$ lies on another square and C coincides with a vertex of the square. What is the area of the entire figure?



DIFFICULT: 5 points each

1 Consider all possible subsets of $\{1,2,3,4,5,6\}$ which do not contain any consecutive numbers. Find the sum of the squares of the products of the numbers in these subsets.

2 In order to complete a large job, 1000 were hired, just enough to complete the job on schedule. All the workers stayed on the job while the first quarter of the work was done, so the first quarter of the work was completed on schedule. Then 100 workers were laid off, so the second quarter of the work was completed behind schedule. Then an additional 100 workers were laid off, so the the third quarter of the work was completed still further behind schedule. Given that all workers work at the same rate, what is the minimum number of additional workers, beyond the 800 workers still on the job at the end of the third quarter that must be hired after three-quarters of the work has been completed so that the entire project can be completed on or before schedule?

3 In $\triangle ABC$, $AB = 7$, $BC = 4$. D is a point on $\overline{AC}$. $BD = 3$ and $\dfrac{AD}{DC} = 2$. Find the area of $\triangle ABC$.

4 The sum $\dfrac{1}{1!9!} + \dfrac{1}{3!7!} + \dfrac{1}{5!5!} + \dfrac{1}{7!3!} + \dfrac{1}{9!1!}$ can be written in the form $\dfrac{2^a}{b!}$, where a and b are positive integers. Find the ordered pair (a, b).

5 Let $n = 29^{31}31^{29}$. How many positive integer divisors of n^2 less than n do not divide n?

VERY DIFFICULT: 8 points each

1 Let P be a point inside equilateral triangle ABC, with $PA = 4$, $PB = 3$, and $PC = 5$. Find the side length of $\triangle ABC$.

2 Let $\text{DARE}(n)$ denote the sum of the digits of n. The first term of a sequence $a_1 = 2009^{2008}$, the second term is $a_2 = \text{DARE}(a_1)$, and the succeeding terms are defined as $a_n = \text{DARE}(a_{n-1})$. Find the value of a_{2008}.

3 In a game of Gobble, letters are randomly arranged in a square. Then you create words by connecting adjacent letters, orthogonally or diagonally, in the proper order. Some examples are the words "dare," "the," and "difficult" as shown:

D			E				F	F		
A				H		I		C	I	
	R	E		T		D		U	L	T

How many ways can we form the word "numbers" using the arrangement shown?

S	S	S	S	S	S	S	S	S	S	S	S	S
S	R	R	R	R	R	R	R	R	R	R	R	S
S	R	E	E	E	E	E	E	E	E	E	R	S
S	R	E	B	B	B	B	B	B	B	E	R	S
S	R	E	B	M	M	M	M	M	B	E	R	S
S	R	E	B	M	U	U	U	M	B	E	R	S
S	R	E	B	M	U	N	U	M	B	E	R	S
S	R	E	B	M	U	U	U	M	B	E	R	S
S	R	E	B	M	M	M	M	M	B	E	R	S
S	R	E	B	B	B	B	B	B	B	E	R	S
S	R	E	E	E	E	E	E	E	E	E	R	S
S	R	R	R	R	R	R	R	R	R	R	R	S
S	S	S	S	S	S	S	S	S	S	S	S	S