EASY: 2 points each

1 In Prof. Cheng's class, 30 students were supposed to take a final exam. However, five were absent and given a score of zero. Including those absent, the class averaged a score of 65 out of 100. What is the largest possible number of perfect-scoring students for this exam?

2 To what number does the series $1+\sqrt{\frac{1}{3}}+\frac{1}{9}+\sqrt{\frac{1}{27}}+\frac{1}{81}+\sqrt{\frac{1}{243}}+\cdots$ converge to?

3 Solve for the radian measure of the smallest positive x such that $\sin(2009x)=\cos(2008x)$.

4 The Sipnayan Core group is thinking about the entrance fee for next year's contest. Being Math enthusiasts, they determined that if the entrance fee is x, then the total number of schools that will join is $\frac{500-x}{x}$% of the schools in the Philippines. How much should they charge to maximize their earnings?

5 The nth term of the Fibonacci sequence is the sum of the previous two terms. If the first two terms of the Fibonacci sequence is 0 and 1, how many prime numbers less than 300 belong to the sequence?

6 The sum of an arithmetic sequence is 2009. The sequence has a common difference $\frac{2}{3}$ and the first term is 25. Find the number of terms in the sequence.

7 If a five-letter word was formed from the letters of the word $YEHEY$, what is the probability that no two adjacent letters are the same?

8 Arrange the following from least to greatest: $\sqrt{5}+\sqrt{3}$, $\sqrt[4]{240}$, $\sqrt{15}$

AVERAGE: 3 points each

1 In a football league of 20 teams, each team plays every other team twice. After the season, Mark's favorite team scored 73 points. Interestingly, the number of wins his team had was equal to the product of the number of losses they had and the difference between their draws and losses. If a team earns 3 points for a win, 1 point for a draw, and nothing for a loss, and Mark's team lost both their games against another team, how many games did Mark's favorite team win?

2 A pyramid of cubes is constructed as follows: the top layer contains one cube, the second layer contains three cubes, the third layer contains six cubes, the fourth layer contains ten cubes, and so on. If we build a pyramid with 50 layers, what is the total number of cubes in the pyramid?

3 Find the sum of all the integral values n can take for the expression $\frac{n^3+4n^2+2n+5}{n+3}$ to be an integer.

4 Adam and Kris play a little banana-based game. Initially, Adam has 201 bananas in his pile, while Kris has 156 bananas in his pile. Every day, each of them eats one banana from his pile if possible, and then simultaneously gives the other half of what they have remaining in their own pile, rounded down. After all bananas are eaten, how many did Kris eat?

5 If the polynomial $1-x+x^2-x^3+x^4-\cdots-x^9+x^{10}$ is expressed as $a_0+a_1y+a_2y^2+\cdots+a_9y^9+a_{10}y^{10}$, where $y=x-1$, what is the value of $a_0+a_1+a_2+\cdots+a_{10}$?

DIFFICULT: 5 points each

1 Davin the Fly is inside a 6 cm by 6 cm by 6 cm box. If he is in the bottom corner, and he can only move 1 cm left, 1 cm right, or fly 1 cm up per step but cannot move back toward his starting point, how many different paths can he take to the upper corner diagonally opposite him if he needs to pass through the center of the box?

2 Let f be a function defined on the set of integers such that $f(1) = 3$ and $f(x)f(y)$ is equal to $f(x+y) + f(x-y)$. What is $f(7)$?

3 A circle rolls along inside the arc of the parabola $y = x^2$. What is the radius of the largest circle that will eventually reach the bottom of the parabola, tangent to the origin, without getting stuck before getting there?

4 A *round number* is an integer whose base 2 representation has at least as many zeros as ones. How many round numbers are there less than or equal to 100?

5 Find values of b and c such that $x = \sqrt{19} + i\sqrt{39}$ is a root of $x^4 + bx^2 + c$.

VERY DIFFICULT: 8 points each

1 If $P(x)$ denotes a polynomial of degree 2008 such that $P(k) = \dfrac{k}{k+1}$ for $k = 0, 1, 2, 3, \ldots, 2008$, determine $P(2009)$.

2 Let S be the set of integers from 1 to 2^{2009} and D be the sum of the greatest odd divisors of each of the elements of S. Find D. Express your answer in the form $\dfrac{a^b + c}{d}$.

3 In the following figure, $AB = 7$, $BC = 6$, and $CA = 5$. The quadrilaterals $ABQP$, $BCSR$, and $CAUT$ are all squares. Find the area of the hexagon $PQRSTU$.