

Sipnayan 2016

Junior High School

November 12, 2016

Inexact wording. I am doubtful that any references to Vincent or Pikachu are in the original. With thanks to Vincent Dela Cruz for providing the questions. If you have any corrections, please contact me at cj@cjquines.com, or through my Facebook account, Carl Joshua Quines.

Written round, two hours

Easy, two points each

1. A bishop could attack other pieces only by moving through the chessboard diagonally. In an 8×8 chessboard, what is the least number of bishops needed so that no matter which square you choose, you will always get attacked by a bishop? [8]
2. Evaluate $\sqrt{\binom{9}{2} + \binom{10}{2} + \binom{40}{2} + \binom{41}{2}}$. [41]
3. Lemy the Fly and Jeric the Ant are on one corner of a unit cube. They are going to the opposite corner of the cube. Lemy the Fly can fly through the interior of the cube, while Jeric the Ant can travel on the faces of the cube. The insects are going to travel the least possible distance they could. How much distance shorter does Lemy the Fly travel than Jeric the Ant? $[\sqrt{5} - \sqrt{3} \text{ units}]$
4. If $ab = 1$, what is the value of $\frac{b}{1 - \frac{1}{a}} + \frac{a}{1 - \frac{1}{b}} + 1$? [0]
5. How many ways could you arrange the letters of the word ABSOLUTE if A can't be next to a consonant? [8640]
6. A quadrilateral *PONG* is inscribed in a circle. If $\angle P = 123^\circ$, what is $\angle N$? [57°]
7. What is the smallest possible surface area of a rectangular prism with integer side lengths and a volume of 60 cubic units? [94 sq. units]
8. The roots of a quadratic equation $ax^2 + bx + c = 0$ are r and s . If r^2 and s^2 are the roots of the same quadratic equation, how many possible pairs (r, s) are there? [3]

Average, three points each

1. A Tetris number is a three-digit number which is equal to the sum of each two-digit number you could make with the digits. One Tetris number is 132, because $132 = 13 + 12 + 31 + 32 + 21 + 23$. What is the sum of all the Tetris numbers? [792]
2. Vincent has some number of chocolate chips, and he arranged them into 8 equal rows. Pikachu ate of the chips and noticed that he could arrange the remaining chips into 9 equal rows. He ate another one and noticed that he could arrange the remaining chips into 10 equal rows. What is the least possible number of chocolate chips Vincent had at the beginning? [352]
3. When you divide $x + 7$ by 6, you get a remainder of 5. When you divide $x + 6$ by 5, you get a remainder of 4. What is the remainder if you divide $x + 8$ by 30? [6]

4. There are 10 people to be seated, yet there are 11 chairs arranged in a line. What is the probability that person A is sitting next to an empty seat? $\left[\frac{2}{11} \right]$
5. If $3x^4 - 7x^3 - 4x^2 - 7x + 3 = 0$, what are the possible values of x ? $\left[3, \frac{1}{3} \right]$

Difficult, five points each

1. If a, b and c are the roots of $f(x) = x^3 - 4x^2 - 5x + 8$, what is the value of $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}$? $\left[\frac{11}{12} \right]$
2. Solon and Noe are playing volleyball. They are both 20 cm from the center and they are on the opposite sides of each other. Solon hits the ball, it reaches the 25 cm mark of Noe's side. Then Noe hits the ball, it reaches the 30 cm mark of Solon's side. The game is continued like this until the ball reaches the 1 m mark. What is the total horizontal distance the ball covered? $[1920 \text{ cm}]$
3. m is a root of $A(x) = x^2 - nx + 10 = 0$, and n is a root of $B(x) = x - mx + 20 = 0$. What is the value of $\left(\frac{n}{m} + \frac{m}{n} \right)^2$? $\left[\frac{25}{4} \right]$
4. There are 3 alarm clocks, red, yellow and blue. The red alarm clock beeps every 3 seconds. The yellow alarm clock beeps every 4 seconds. The blue alarm clock beeps every 5 seconds. In the interval of 600 seconds, and the alarm clocks and turned on at the same time, how many times did only 2 alarm clocks beep at the same time? $[90]$
5. The 3rd and 13th terms of an arithmetic sequence are the same as the 4th and 6th terms of a geometric sequence respectively. The 8th term of the arithmetic sequence is 25 and the 5th term of the geometric sequence is 20. What are the possible first terms of the arithmetic sequence? $[4, 46]$

Very difficult, eight points each

1. Points P, A , and U are collinear, in that order. Squares $PIKA$ and $ACHU$ are constructed on the same side of line PU . If $PK = KH = \sqrt{2}$, what is the area of triangle PKH ? $\left[\frac{1}{2} \text{ sq. unit} \right]$
2. Box A and B contain red and blue marbles. They have the same total number of red and blue marbles combined. Matt randomly chooses a marble from box A and B , and does not return them. Kim does the same. The probability that Matt draws a red marble from box A and a blue marble from box B is $\frac{1}{8}$. The probability that Kim draws a red marble from box A and a blue marble from box B is $\frac{3}{7}$. If there are 2 red marbles in box A , how many blue marbles are there in box B ? $[4]$
3. David likes primes and almost primes. An almost prime is a number whose product of its digits is prime. How many numbers less than 1000 are prime or almost prime? $[178]$