Semifinal Round A

Easy: 2pts, 30s

SFA E-1 The first two terms of an arithmetic sequence are $\sqrt{3}$ and $\sqrt{27}$. If the 10th term is $\sqrt{m}$ where m is a real number, find m. [1083]

SFA E-2 What is the smallest angle formed by the hour hand and the minute hand of an analog clock at exactly 9:15 PM? [172.5°]

SFA E-3 If x and y are real numbers such that $\frac{2x}{y} + \frac{3y}{x} - \frac{4}{y} - \frac{12}{x} = -\frac{14}{xy}$, find $x + y + xy$. [5]

SFA E-4 How many digits are used in writing the integers from 1 to 10^6 inclusive? [5 888 896]

SFA E-5 How many three-digit positive integers are there such that the sum of any two of its digits is even? [225]

Average: 3pts, 60s

SFA A-1 Find the minimum value of $(2x + 3y)\left(\frac{8}{x} + \frac{3}{y}\right)$ given that x and y are positive. [49]

SFA A-2 How many positive integers less than equal to 2016 are divisible by 2, but are neither divisible by 3 nor by 7? [576]

SFA A-3 A4 paper has the special property of retaining the proportion of its sides after being folded in half crosswise. Assuming that the shorter side (i.e. width) of an A4 sheet of paper is 1 unit, what is the area of a circle circumscribed around the sheet *in square units*? $\left[\frac{3\pi}{4}\text{ units}^2\right]$

SFA A-4 The number $x = 122333444455555\cdots20202020$ is formed by appending one 1, two 2's, three 3's, and so on, until twenty 20's. Find the remainder when x is divided by 9. [8]

SFA A-5 Suppose P is a point in the interior of rectangle $MATH$. If $PM = 5\,\text{cm}$, $PA = 7\,\text{cm}$, $PT = 9\,\text{cm}$, what is PH? $\left[\sqrt{57}\,\text{cm}\right]$

Difficult: 5pts, 90s

SFA D-1 Let the sum of all integers n such that $\frac{2n^2 + 9}{n + 3}$ is an integer be A. What is $|A|$? [24]

SFA D-2 The digits of the numbers 0 to 99 are written as a continuous string and scrambled to form another string of digits. The number of distinct rearrangements of the string such that 10 consecutive zeroes appear in the scrambling is of the form $\frac{a!}{b!^c}$, where a, b, and c are positive integers. Find $a + bc$. (*Non-distinct* rearrangements have the same digits in the same order.) [361]

SFA D-3 Consider a regular hexagon $ABCDEF$ with sides 1 unit long. The two equilateral triangles $\triangle ACE$ and $\triangle BDF$ are constructed. What is the area of the hexagon whose six vertices are the six points at which ACE and BDF meet? $\left[\frac{\sqrt{3}}{2}\text{ units}^2\right]$

SFA D-4 If $x + \frac{1}{x} = 3$ and $x^5 + \frac{1}{x^2} = 9$, find $(x^4 + x)^2$. [36]

SFA D-5 The three interior angles of an acute triangle measure $a°$, $b°$, and $c°$, where a, b, c are positive integers such that $a < b < c$. Find the smallest possible value of b. [46]

Semifinal Round B

Easy: 2pts, 30s

SFB E-1 Mandybis traveling indefinitely in a straight line at a geometric progression. Today, she travels 15 miles. Tomorrow she plans to travel 12.5 miles, then $10\frac{5}{12}$, and so on, How far will she be at the end of her journey? [90 miles]

SFB E-2 What is the sum of the coefficients of $P(x) = 3(x+5)^3 + 2(x+6)^2 + (x+7)$? [754]

SFB E-3 A group of friends went for lunch and they shared the bill equally. There would be a shortage of $5 if each one paid $16, and there would be an excess of $17 if each one paid $17. How many dollars was the bill? [$357]

SFB E-4 Team Sonic and Team Tails play in a series. The first team to win three games wins the series. The game ends when someone wins the series. Each team is equally likely to win each game, there are no ties, the outcomes of the indicidual games are independent. If Tails wins the second game and Sonic wins the series, what is the probability that Tails wins the first game? $\left[\frac{1}{5}\right]$

SFB E-5 Find the largest positive integer x such that $3^{2048} - 1$ is divisible by 2^x. [13]

Average: 3pts, 60s

SFB E-1 Suppose that x, y and z are nonnegative real numbers. If $\frac{x+y}{3} = \frac{y+z}{4} = \frac{z+x}{5}$, find the ratio $x : y : z$. [2 : 1 : 3]

SFB E-2 Jay and Dee are playing on a seesaw that is 200 cm long with its fulcrum exactly at the center. The seesaw becomes perfectly balanced when Dee sits 25 cm from the edge and Jay sits 30 cm from the fulcrum. If Dee remains stationary and the fulcrum is moved 50 cm away from Jay, how far must Jay now sit from the edge to balance the seesaw once more? [60 cm]

SFB E-3 Michael jogged along a path. He put down a piece of biscuit after every 10 steps from the starting point. When Michael put down the nth biscuit, his dog reached the starting point of the path and walked along the same path at twice Michael's speed. It also ate up all the biscuits om the way. When Michael put down the 60th biscuit he found that the dog had just eaten the nth biscuit, Assuming that the amount of time the dog takes to eat a biscuit is negoigible, find n. [40]

SFB E-4 Pac-Man is a video game character that is formed by removing a sector with a specified angle to form its "mouth". Suppose that Pac-Man's mouth is opened at 60°, and that the sector that is removed has the same area as an equilateral triangle. The ratio of the radius of the sector that was removed to the side of the triangle can be expressed in the form $\sqrt[4]{u}$, where u is a real number in lowest terms. What is u? $\left[\frac{27}{4\pi^2}\right]$

SFB E-5 Minnie, Maxie, and Chang have constant rate of solving a hard math question. Ehen Minnie and Chang work together, it takes 1 hour and 2 minutes to solve a had math question. When Chang and Maxie work together, it takes 2 hours, When Minnie and Maxie work together and Chang constantly interrupts them, it takes 3 hours for them to solve. Assuming that the rate of interruption is the same as the rate of solving a hard math problem, how many hours would it take for Minnie and Maxie to solve the question without Chang interrupting? [1.5 hrs]

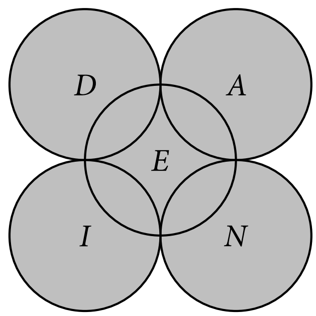
Difficult Round: 5pts, 90s

SFB D-1 Ivan and his 10 brothers, along with n girls, caught a total of $n^2 + 9n - 2$ jellyfish, with each of them catching the same number of jellyfish. Find n. [9]

SFB D-2 What is the maximum number of acute angles in a 20-sided polygon? [14]

SFB D-3 Let x and y be rational such that $x^2 - y = 4$ and $y^2 - x = 8$. What is the integral value of $(x^5 - y^5) - (x^3 - y^3)$? [216]

SFB D-4 Four congruent circles D, A, N, and I are tangent to each other such that D is tangent to circles A and I, A is tangent to circles D and N, and N is tangent to A and I. Out of boredom, Jerome draws a circle that intersects the circles at their points of tangency and called the circle E. He discoveed that circle E had an area of 144π cm^2. Find the area of the shaded region. [$(576 + 432\pi)$ cm^2]



SFB D-5 A *Himig number* is a positive integer with at least 3 and no more than 5 distinct prime divisors. How many Himig numbers less than 100 are there? [8]

Final Round

Easy: 2pts, 30s

F E-Tetris Twelve fair dice are rolled. What is the probability that the product of the numbers on the top faces is prime? Express your answer in the form $\frac{1}{a^b}$, where a and b are positive integers. $\left[\frac{1}{6^{10}}\right]$

F E-Pong By starting with the sequence $1, 2, 3, 4, \ldots$ and replacing each term n by fve terms $n, n+1, n+2, n+3, n+4$, the following new sequence is obtained:

$$1, 2, 3, 4, 5, 2, 3, 4, 5, 6, 3, 4, 5, 6, 7, 4, 5, 6, 7, 8, \ldots$$

What is the 2016th term of the new sequence? [404]

F E-Pac-Man If $\frac{12}{2015} < \frac{m}{n} < \frac{13}{2016}$, where m and n are positive integers, find the smallest possible value of n. [156]

F E-Invaders Define a *space invader* to be a positive integer whose digits are all distinct, and such that each digit is a factor of the number. Find the greatest possible four-digit space invader. [9864]

F E-Sonic What is the remainder when the sum of all even numbers from 1 to 1000 inclusive is divided by 13? [3]

Average: 3pts, 60s

F A-Tetris Let a, b and c be positive reals such that $a+b+c=15$ and $\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}=\frac{21}{23}$. What is $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$? $\left[\frac{246}{23}\right]$

F A-Pong Find the sum of all the integers between 150 and 750 that leave a remainder of 6 when divided by 10. [27 060]

F A-Pac-Man If $a36\,405\,489\,812\,706\,44b$ is divisible by 99, what are the possible values of ab? [9 and 1]

F A-Invaders How many ways can you arrange the letters of *PANORAMA* such that no two vowels are beside each other? [480]

F A-Sonic Given that ABC is an isosceles right triangle with $\angle CAB = 90^\circ$ and $AB = AC = 48$. A square with area S has all its four vertices lying on the sides of triangle ABC. Find the sum of all possible values of S. [1088]

Difficult: 5pts, 90s

F D-Tetris Let A_n be the set containing all elements that can be expressed as x^n, where $1 \le x^n \le 100$ and $x, n \in \mathbb{Z}^+$, and $f(A^n)$ be the sum of all the elements of A_n for a given n. Given that the function value of $f(A_n)$ is 0 if A_n has less than two elements in its set, find $\sum_{n=1}^{100} f(A_n)$. [5731]

F D-Pong Two sides of a square are tangent to a circle with diameter 8. One corner of the square lies on the circle. There are positive integers m and n so that the of the square is $m+\sqrt{n}$. Find $m+n$. [536]

F D-Pac-Man A circle with area 40 is tangent to a circle with area 10. Let R be the smallest rectangle containing both circles. The area of R can be expressed as $\frac{n}{\pi}$ units^2. Find the area of a circle with circumference n. $\left[\frac{6400}{\pi} \text{ units}^2\right]$

F D-Invaders The sum of 2016 consecutive positive integers is the square of an integer. What is the minimum value of the largest of these integers? [2019]

F D-Sonic For positive integers m and n, define its *HIGH SCORE* to be the sum of the numerator and denominator when $\frac{m}{n}+\frac{n}{m}$ is written as a fraction in lowest terms. For example, since $\frac{3}{2}+\frac{2}{3}=\frac{13}{6}$, the HIGH SCORE of 2 and 3 is $13+6=19$. What is the largest prime factor of the HIGH SCORE of 24^2 and 26^2? [157]

Very Difficult: 8pts, 120s

F VD-Tetris Let m and n be positive integers. If $m+2$ is a multiple of n while $n+2$ is a multiple of m, find the sum of all possible values of n. [16]

F VD-Pong A lamb is attached at the end of a rope with length 9 meters. The other end of the rope is tied to one corner of a fence. The fence is an equilateral triangle with side length 2 meters. The lamb travels clockwise. If the rope remains taut (doesn't break or strech) whole the sheep is traveling, what is the maximum distance the lamb would have traveled before reaching a full stop? $\left[\frac{50\pi}{3} \text{ meters}\right]$

F VD-Pac-Man Let a and n be positive integers such that $a+(a+1)+(a+2)+\ldots+(a+n)=100$. Find the sum of all possible values of n. [11]

F VD-Invaders Two legs of a right triangle, which are also two of the altitudes of the triangle, have lengths $6\sqrt{3}$ and 4. What is the length of the third altitude? $\left[\frac{12\sqrt{93}}{31}\right]$

F VD-Sonic If the greatest real root to the equation $\left(4+\sqrt{3}\right)x^2-\left(5+6\sqrt{3}\right)x+\left(1-\sqrt{3}\right)=0$ can be simplified as $a+b\sqrt{3}$, where a and b are rational, find $a+b$. [2]