Semifinal Round A

Easy: 2pts, 30s

SFA E-1 A positive divisor of 10^{10} is chosen at random. What is the probability that it is prime? $\left[\dfrac{2}{121}\right]$

SFA E-2 For what real values of y will the equation $\dfrac{m+2015}{m+2017} = \dfrac{m+y}{m+2019}$ have no solution for m? $[2017, 2019]$

SFA E-3 Woody and Bo Peep each scretly chose a random integer in the set $\{0, 1, 2, \ldots, 100\}$, then multiplied their chosen numbers together. What is the probability that the result is even? $\left[\dfrac{7701}{10\,201}\right]$

SFA E-4 Let $x = a + b$, $y = a - b$, and $z = ab$, where a and b are real numvbers. Find all pairs (a, b) that satisfy $x(y^2 + z) + y(x^2 - z) = 250\,000$. $[(50, b) \text{ where } b \in \mathbb{R}]$

SFA E-5 Let $ABCD$ be a square, and let E be the midpoint of segment BC. Construct a circle cenered at A that passes through B and D, and a circle centered at E that passes through B and C. These two circles intersect at two points; one of them is B, and name the other point of intersection as F. Find the measure of $\angle AFE$. $[90^\circ]$

Average: 3pts, 60s

SFA A-1 In a convex n-sided polygon, the measure of all but two of the interior angles of the polygon is equal to 160°. What is the sum of all possible values of n? $[187]$

SFA A-2 Find the last three digits of 998^{11}. $[952]$

SFA A-3 How many 1000-character strings are there that only consist of vowels (a, e, i, o, and u) and no two adjacent letters in the string are the same? Express your answer as a product of prime powers. $[2^{1998} \times 5]$

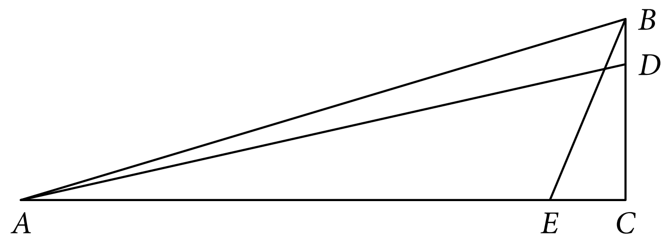
SFA A-4 Suppose x integers are chosen without repetition from the set $\{1, 2, 3, \ldots, 2019\}$. If the proabilty that there are at least two elements from the chosen integers whose difference is 19 is 100%, what is the least value of x? $[1013]$

SFA A-5 Let equilateral triangle XYZ with side length 9 be inscribed in a circle and let A be on minor arc XY such that $AX \times AY = 14$. Find the length of AZ. $[\sqrt{95}]$

Difficult: 5pts, 90s

SFA D-1 Find the difference between the maximum and minimum values of $f(x) = |x+3| - |2x-6| + |x-4|$. $[12]$

SFA D-2 In the diagram below, BC is perpendicular to AC. D is a point on BC such that $BC = 4BD$. E is a point on AC such that $AC = 8CE$. If $AD = 41$ cm and $BE = 13$ cm, determine the length of AB. $[4\sqrt{109}\text{ cm}]$



SFA D-3 How many positive integers less than 2019 have a digit sum of 10? [120]

SFA D-4 Bo Peep, Billy, Goat, and Gruff are playing a dart game at the carnival. There is a square grid split into 11 rows, each with 11 columns, totalling 121 cells. Each of them throws a dart which will land in a random cell — assume that it is equally likely for each dart to end up in any of the cells, each player's dart is unaffected by the other darts, and it is possible for more than one dart to land in the same cell. If their four darts land in four different cells, and the four cells that were hit form the shape of an axis-aligned retctangle, they win! What is the probability that they win the game? Express your answer in lowest terms and in the form $\dfrac{a}{p^n}$, where p is prime. $\left[\dfrac{600}{11^6}\right]$

SFA D-5 In triangle ABC, $AB = x + 3y$, $BC = x + 2y$, and $CA = x + y$, where x is a positive integer and $0 < y < 2$. If the length of the altitude to side BC is equal to x, how many possible values of x are there? [23]

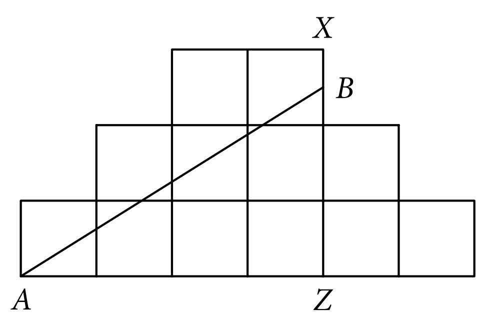
Tiebreaker: 5min

SFA TB-1 Find the second largest positive integer n such that $(n - 1)^3$ is divisible by $n + 8$. [235]

Semifinal Round B

Easy: 2pts, 30s

SFB E-1 In the dodecagon composed of 12 unit squares below, the line AB divies the figure into two regions with areas 4 and 8. Find the value of $\dfrac{XB}{BZ}$. $\left[\dfrac{1}{5}\right]$



SFB E-2 In a math competition called IPSNAYAN, the average of the participants is 14 years old, while the average age of the coaches is 35 years old. Taken together, the average age of both groups is 18 years old. What is the ratio, in lowest terms, of number of participants to number of coaches? [17 : 4]

SFB E-3 Any of Duck and Bunny's plans have a 75% chance to be absolute horrible, and a 25% chance to be somewhat okay. If they pitch 5 plans to Woody, what is the probability that at least one of them isn't absolutely horrible? $\left[\dfrac{781}{1024}\right]$

SFB E-4 Let n be a three-digit number, and n' be the number obtained by reversing the digits of n. Find all n such that $n + n' = 524$. $[163, 262, 361, 460]$

SFB E-5 Let a, b, and c be positive real numbers such that $a = 2b^2 = 5c^3 = abc$. What is the value of abc? $\left[\sqrt[5]{200}\right]$

Average: 3pts, 60s

SFB A-1 Let N be the least common multiple of the first 15 positive integers. What are the last three digits of N? $[360]$

SFB A-2 Suppose $a_1, a_2, a_3, \ldots, a_{19}$ are the roots of the equation $x^{19} - 20x - 19 = 0$. Find the exact value of $\sum_{i=1}^{19} a_i^{19}$. $[361]$

SFB A-3 There are 2017 men, numbered from 1 to 2017, and 2019 women, numbered from 1 to 2019. Man i can only be paired with woman j if $i \leq j$. If each woman can only be paired with at most one man, in how many ways can each man be paired with exactly one woman? $[3^{2017}]$

SFB A-4 Given that $\sin 2\theta = -\frac{1}{5}$, find $\sin^6\theta + \cos^6\theta$. Express the answer in lowest terms. $\left[\dfrac{97}{100}\right]$

SFB A-5 A cone may be created from a semicircular sheet of paper by joining the ends together. What is the volume of the cone formed if the sheet of paper has radius 10? $\left[\dfrac{125\sqrt{3}\pi}{3}\right]$

Difficult: 5pts, 90s

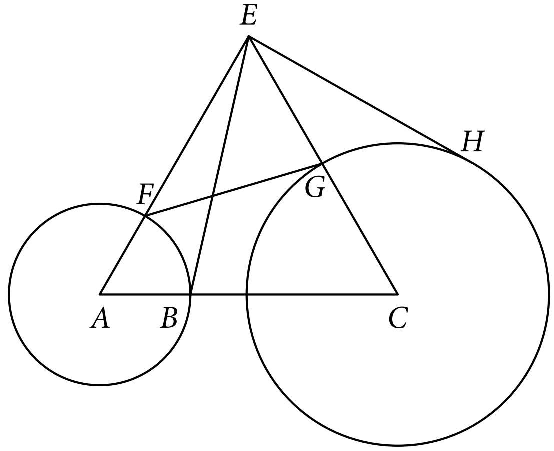
SFB D-1 Find all possible ordered pairs (X, Y) such that the number $7\,2X3\,640\,548\,981\,270\,644\,Y72$ is divisible by 99. $[(4, 6)]$

SFB D-2 In a circle, there are three parallel chords of length 16, a, and b, respectively, where $a, b < 16$. The distance between the two chords of length 16 and a is 2 units while the distance between the two chords of length 16 and b is 22 units. If $a = b$, what is the value of a? $[4\sqrt{5}\text{ units}]$

SFB D-3 Define $\lfloor x \rfloor$ to be the greatest integer less than or equal to x. Find the number of positive integers $n < 2525$ such that $n = x\lfloor x \rfloor$. $[1250]$

SFB D-4 Woody, Buzz Lightyear, Mr. Potato Head, Rex, and six identical Aliens are to be seated around a circular table. Given that each Alien must be seated next to at least one other Alien, how many ways can they be seated around the table? $[264]$

SFB D-5 In the figure below, Circles X and Y with centers A and C have radii 6 and 10 respectively. E is a point outside the two circles such that triangle EAC is equilateral. EH is drawn such that it is tangent to circle Y and $EH = 17$. AE intersects circle X at F, CE intersects circle Y at G, and line AC intersects circle X at B. Find the value of $8EB^2 - 5EH^2 - 3FG^2$. $[560]$



Final Round

Easy: 2pts, 30s

F E-Woody If $x = \left(4 + \dfrac{1}{\sqrt[3]{65^2} + \sqrt[3]{4160} + \sqrt[3]{8^4}}\right)^6$, find the value of x. [4225]

F E-Buzz An equilateral triangle has side length x and height y. Find $x^2 : y^2$. [4 : 3]

F E-Hamm It is known that the equation $x^2 - ax + 107b = 0$ has distinct, positive, integral roots. Given that a and b are prime numbers, what is the value of $a + b$? [111]

F E-Potato Duke Caboom claims that if he flips a fair coin n times and rolls a fair six-sided die 5 times, it is more likely for his die to land on a number greater than 2 five times in a row than it is for his coin to land on Heads for all n times. Find the minimum value of n for which his claim is true. [3]

F E-Forky If $a \heartsuit b = \dfrac{ab}{\sqrt{a^2 + b^2}}$, what is $\left(\left(\dfrac{1}{\sqrt{1}} \heartsuit \dfrac{1}{\sqrt{7}}\right) \heartsuit \left(\dfrac{1}{\sqrt{3}} \heartsuit \dfrac{1}{\sqrt{5}}\right)\right) \heartsuit \left(\left(\dfrac{1}{\sqrt{2}} \heartsuit \dfrac{1}{\sqrt{8}}\right) \heartsuit \left(\dfrac{1}{\sqrt{4}} \heartsuit \dfrac{1}{\sqrt{6}}\right)\right)$? $\left[\dfrac{1}{6}\right]$

Average: 3pts, 60s

F A-Woody Consider the set of all ordered quintuples of integers $(a_1, a_2, a_3, a_4, a_5)$, where each element satisfies $1 \le a_i \le 7$. How many such quintuples satisfy $a_1 < a_2 < a_3$ and $a_3 > a_4 > a_5$? [371]

F A-Buzz Let θ be an angle such that $\sec^4\theta - \tan^4\theta = 2019$. What is the value of $\sin^2\theta$? $\left[\dfrac{1009}{1010}\right]$

F A-Hamm How many integers $k \le 2019$ are there such that $3^a + 3^b + 3^c = k$, where a, b, and c are integers? [63]

F A-Potato Let x and y be real numbers such that $x^2 - 6x - 15 = 0$ and $y^2 - 6y - 15 = 0$. Find all possible values of $x^3 - y^3$. $[0, \pm 204\sqrt{6}]$

F A-Forky Gabby Gabby has 20 identical coins that she will give to 6 of her best friends, Benson, Banson, Binson, Bonson, Bunson, and Forky. If each of her friends must receive at least one coin and one of her friends should receive at least 10 coins, how many ways can she distribute the coins among her friends? [1512]

Difficult: 5pts, 90s

F D-Woody For how many integer values of a does the equation $y = \prod_{b=1}^{100}(x^2 - bx + a)$ intersect the x-axis exactly 50 times? [37]

F D-Buzz Call a finite sequence of integers a *mountain permutation* if there exists some positive integer n such that the sequence is a permutation of the first n natural numbers, and for some positive integer k, the first k terms are increasing and the last $n - k + 1$ terms are decreasing. For example, 1, 3, 4, 2 and 5, 4, 3, 2, 1 are mountain permutations while 1, 4, 6, 5, 2 and 4, 3, 2, 1, 5 are not. How many mountain permutations of length 2019 are there? $[2^{2018}]$

F D-Hamm Given that x, y, and z are non-negative real numbers and $\frac{xy^2z^3}{3} + 3 = 2019$, minimize the value of $x + y + z$. $[6\sqrt[6]{56}]$

F D-Potato What is the smallest five-digit positive integer whose digits are distinct and that is divisible by each of its digits? [12 384]

F D-Forky Find the product of the minimum and maximum values of $a + b + c$, where a, b, and c satisfy $a^3 + b^3 + c^3 + 3b^2c + 3bc^2 + 9ab + 9ac = 27$. [−18]

Very Difficult: 8pts, 120s

F VD-Woody Woody and his GANG GANG are on an adventure to save their friend Buzz Lightyear. However, they would need access to Buzz's computer which sould give out the exact location to Buzz's Space Ranger Armor. They need to enter his password and his computer shows the following text: "The password is '$BUZZBUZZ$' which is an 8-digit number. B, U, and Z are distinct digits. This number also has exactly 4 distinct prime divisors. Lastly, this 8-digit number is the second such number when all possible passwords are arranged from least to greatest." What is the password? [10 551 055]

F VD-Buzz Buzz, Woody, and Lotso are shooting free throws. They take turns with Buzz shooting free throws first, followed by Woody, and then, by Lotso. A player's turn ends when he fails to shoot the free throw. Buzz has a 50% chance of successfully shooting a free throw, Woody has a 40% probability of successfully shooting a free throw, while Lotso only has a 25% probability of successfully shooting a free throw. The game ends when someone successfully shoots the free throw two consecutive times. What is the probability Woody wins the game? $\left[\frac{192}{655}\right]$

F VD-Hamm Suppose two cubic polynomials $f(x)$ and $g(x)$ satisfy the following: $f(2) = g(4)$; $f(4) = g(8)$; $f(8) = g(16)$; $f(16) = g(32) + 64$. What is the value of $g(128) - f(64)$? [−9920]

F VD-Potato Mr. Potato Head has an unlimited number of green coins worth 1 peso each, red coins worth 2 pesos each, blue coins worth 3 pesos each, and gold coins also worth 3 pesos each. Suppose Mr. Potato Head wants to buy a drink worth 10 pesos in a vending machine. In how many ways can Mr. Potato Head place his coins into the vending machine if a different ordering of colors is considered a distinct way? [585]

F VD-Forky Let x_1, x_2, x_3 be the real roots of the equation $\sqrt{2019}x^3 - 6058x^2 + 3 = 0$ where $x_1 < x_2 < x_3$. Find the exact value of $2019x_2(x_1 + x_3)$. [6057]